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# Practical Estimation of Losses in Tee Network Antenna Tuning Units

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This article defines the loss mechanisms in antenna tuning units and describes the mathematical basis for a computer program that estimates losses for Tee matching networks

Tee matching networks are widely used in antenna tuning units (ATUs) used by radio communicators in the MF (0.3 to 3.0 MHz) through UHF (0.3 to 3.0 GHz) spectrum. This article describes a computer software tool for estimating the power losses of such networks in general impedance matching applications. The mathematical basis for the program is reviewed, and the results of several illustrative case studies are reported.

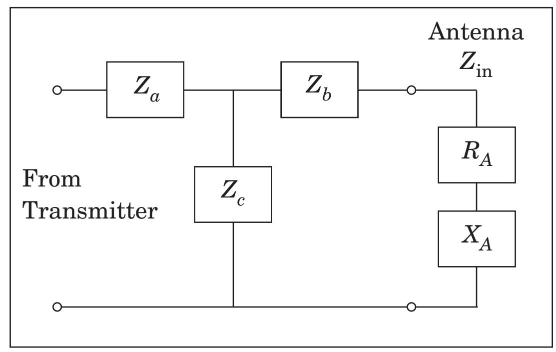


**Figure 1 · Tee network Antenna Tuning Unit.**

## Introduction

The Tee network ATU, shown in Figure 1 with antenna feedpoint load $Z_{in} = R_A + jX_A$, is a classic approach to antenna impedance matching in radio transmission systems. ATU power loss can be a significant consideration in link budget analysis, and obtaining loss estimation values for intelligent planning and design has been historically problematic.

Concerns about ATU losses largely originated with frequency agile HF radio systems, which employ nonresonant or electrically small antennas in various fixed, portable and mobile applications. However, the loss issue is even more relevant and important in higher-frequency mobile and portable wireless products using reduced size antennas. These smaller size antennas may exhibit feedpoint impedances that vary widely as the units move around in complex operating environments. The results of this study provide a useful computational aid for margin prediction and improved system reliability in all systems employing Tee network ATUs.

For discussion here, the coaxial line from the transmitter is assumed to be of characteristic impedance $Z_0 = Z_{line} = 50$ ohms. Further, it is assumed that all network voltages and currents are specified with RMS values.

If the load (that is, antenna feedpoint) is purely resistive with value $R_A$, a λ/4 transmission line section of characteristic impedance $Z_{xfmr} = (Z_0 \cdot R_A)^{1/2}$ will produce the desired impedance match at the design frequency. The Tee network components, idealized for the lossless case, are then pure reactances:

$$|Z_a| = |Z_b| = |Z_c| = \sqrt{Z_0 \cdot R_A} \tag{1}$$

[1] with $Z_c$ of opposite sign from $Z_a$ and $Z_b$; this results in a lumped element circuit equivalent to a λ/4 section of transmission line with the appropriate characteristic impedance. Interested readers can find more complete background in [2], where it is shown that the impedance matrix for a section of lossy transmission line with propagation constant $\gamma = \alpha + j\beta$ and length $d$ is:

$$Z = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} Z_0 \coth \gamma d & \dfrac{Z_0}{\sinh \gamma d} \\ \dfrac{Z_0}{\sinh \gamma d} & Z_0 \coth \gamma d \end{bmatrix} \tag{2}$$

Simplifying to the lossless case,

$$Z_a = Z_b = jZ_0 \tan \beta \frac{d}{2}; \quad Z_c = -jZ_0 \csc \beta d \tag{3}$$

With $\beta = 2\pi/\lambda$ and $d = \lambda/4$ the results given in [1] follow directly.

Figure 2 shows the Tee network ATU circuit with greater detail, for the case of inductive input and output legs and a capacitive shunt leg reactance. It is also possible to have the input/output legs capacitive with the shunt leg inductive as an alternative, but the Figure 2 configuration is preferred for use with radio transmitters because its lowpass filter behavior attenuates the harmonic output produced (to varying degrees) by all high power RF amplifiers.

For the moment, the premise is continued that $X_A = 0$, so the antenna $Z_{in} = R_A$ is purely resistive. Analysis of the Figure 2 network with conventional circuit theory to obtain voltage and current expressions is then straightforward. However, the results are of limited utility because, at this point, the reactances are lossless and the load purely real.

## Extension to Complex Load

Generally, $X_A \neq 0$ and antenna feed $Z_{in} = R_A + jX_A$. In this case, the standard practice is to use the antenna feedpoint reactance $jX_A$ to make up part of $Z_b = jX_2$ from Figure 1, with total value $X_2$ calculated according to Eq. (1). Hence, the actual reactance placed in output leg "b" of the ATU is:

$$X'_2 = X_2 - X_A \tag{4}$$

## Extension to Lossy Reactances

With the above procedure of routinely incorporating $X_A$ into the ATU's output leg reactance $Z_b = jX_2$, the impedance matching task is reduced to matching a real load to a real transmission line characteristic impedance $Z_0$, which has been specified to be 50 ohms throughout this discussion. Denoting inductance $Q$-factor by $Q_L$ and capacitor $Q$-factor by $Q_C$ according to the most fundamental specification of $Q$:

$$Q = \frac{\text{reactance in ohms}}{\text{resistance in ohms}} \tag{5}$$

allows the calculation of lossy reactive element resistances through:

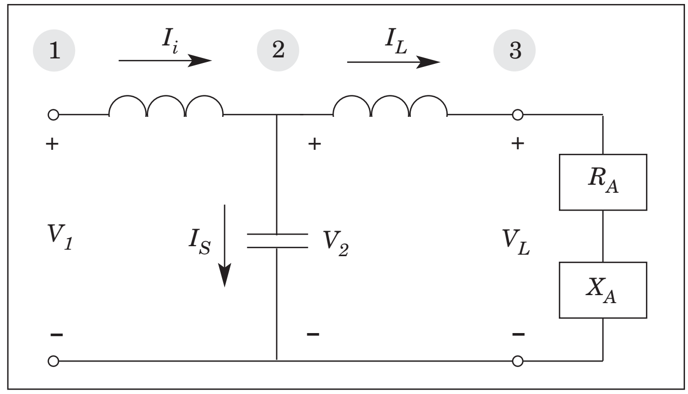


**Figure 2 · A more detailed ATU circuit diagram.**

$$R_L = R_{ind} = \frac{jX_L}{Q_L} \text{ and } R_C = R_{cap} = \left| \frac{-jX_C}{Q_C} \right| \tag{6}$$

In Figure 3, a dashed line appears through the block previously occupied by $X_A$ to represent replacement by a short circuit connection, $jX_2$ in the ATU output leg has been changed to $jX'_2$ to indicate incorporation of $X_A$ into $X_2$, and the three lossy reactance resistances are denoted by R1, R2, and R3.

The introduction of component losses requires a more robust solution strategy, as the application conditions for Eq. (1) are now violated, and its guidance is now potentially highly unreliable and inaccurate. An analytical attempt at solution of the new, real-world problem quickly becomes egregiously heinous, and a computer based numerical solution is highly preferable.

## Program Equations and Strategy

Even in the lossy element case, it remains practical to readily obtain 1:1 SWR at the connection of the transmitter output coax to the ATU input for the vast majority of, if not for all, complex antenna $Z_{in}$ impedances. However, as practical radio communicators know, obtaining a matched impedance condition now generally is an experimental adjustment procedure under human operator or

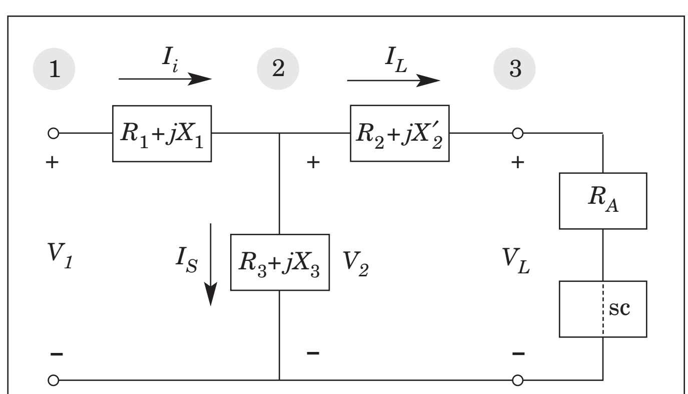


**Figure 3 · ATU circuit elements with losses included.**

microprocessor control, monitoring input SWR value as ATU reactance values are varied.

The computer program objectives are (1) given $Q_L$ and $Q_C$, determine reactance values $X_1$ - $X_3$ that will give a matched impedance condition between antenna and coaxial RF feed, and (2) determine the percent power dissipated in each of the three ATU legs, as well as in the antenna feed resistance $R_A$. Note that $R_A$ is actually a series combination of ohmic loss and radiation resistance, and separation of the two is beyond the scope of this study. The reader should note that, for this particular study, all ATU inductors are assumed to have the same $Q_L$, and all capacitors are assumed to have the same $Q_C$.

Since an accounting for the percent distribution of RF input power is sought, the numerical value of input power $P_{in}$ is immaterial, and is arbitrarily set to 100 watts in the code. Relations for the network voltages and currents are developed below.

Refer to Figure 3, recalling Eq. (4) shows $X_2 = X'_2 + X_A$ and keeping in mind that $X_A$ physically is in the antenna feedpoint load. Assume (*i*) a matched condition to $Z_0 = 50$ is achieved, (*ii*) input power $P_{in}$ is specified, (*iii*) RMS values of voltage and current are used, and (*iv*) the ATU shunt leg is capacitive while the input and output legs are inductive. By Ohm's Law,

$$P_{in} = I_i^2 Z_0 = \frac{V_i^2}{Z_0} \Rightarrow I_i = \sqrt{\frac{P_{in}}{Z_0}} \text{ and } V_i = \sqrt{P_{in} \cdot Z_0} \qquad (7)$$

Then, by current division,

$$I_S = I_i \frac{(R_2 + R_A + jX_2)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \qquad (8)$$

and,

$$V_2 = (I_S)(R_3 - jX_3) \qquad (9)$$

Applying current division again, this time to the output leg feeding the antenna:

$$I_L = I_i \frac{(R_3 - jX_3)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \qquad (10)$$

and,

$$V_L = (I_L)(R_A - jX_A) \qquad (11)$$

Computed $Z_{incalc}$ at the ATU input is from (12):

$$Z_{incalc} = (R_1 + jX_1) + \frac{(R_2 + R_A + jX_2)(R_3 - jX_3)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \qquad (12)$$

Because the component resistances are now incorporated into the equations and those resistances, in turn, depend on the corresponding component reactances, the cause for analytical difficulty and need for numerical aid is apparent.

As noted earlier, Eq. (1) is inaccurate and unreliable in the lossy case with significant antenna mismatch, but it does provide a useful initial estimate for the three ATU reactances. MATLAB® [3] includes an optimization function *fminsearch.m* in its Optimization Toolbox library which can be employed to find the minimum of an unconstrained multivariable function $\min_x f(x)$, where $x$ is a vector and $f(x)$ is a function that returns a scalar. The "multivariable" values to be optimized are those for $X_1$, $X_2$, and $X_3$, and the returned scalar is the absolute value of the difference between the desired 50 ohm input and the calculated ATU input impedance at each iteration of the reactance values. For each iteration,

$$R_1 = \left|\frac{X_1}{Q_L}\right| \text{ or } R_1 = \left|\frac{X_1}{Q_C}\right| \qquad (13)$$

depending on whether $X_1$ is inductive/positive or capacitive/negative for that particular iteration. Similar arithmetic is also applied for $R_3$ and $R_2$, noting that $X'_2$ and not $X_2$ is the numerator for calculating $R_2$ because the primed value is that actually placed in the ATU output leg. Note also that although we are starting with a network with positive input and output leg reactances and a negative shunt leg reactance, the matching optimization routine may occasionally change the sign of one or more of the components.

Default values of $Q_L$ and $Q_C$ are set in the computer tool to 100 and 1000, respectively, but the user is prompted and offered the opportunity to change either value when the program is executed. The code essentially implements the following sequence:

- input $Z_{in}$
- accept default $Q_L/Q_C$ or change
- get initial $X$ estimates from Eq. 1
- get $R$ values from $X$s and $Q$
- compute $Z_{incalc}$
- optimize $X_1$, $X_2$, $X_3$ for match to $Z_0$ using *fminsearch* routine
- compute final, optimized $Z_{incalc}$
- compute voltages and currents
- compute power dissipated in ATU components and power delivered to antenna

Optimized ATU input impedances are not generally *exactly* $50 + j0$ ohms, but are so close that reflected power from the ATU input port is insignificant.

## Illustrative Results

The results of five cases with different antenna impedances are shown in the tables below, continued on the following page

In Case 4, the computer tool result of 5.0% power delivered to the antenna agrees with the tabulated value on p. 68 of [4]. Note that Case 4 was also run with $Q_L$ = 400 for comparison with [4]. The full table of results is omitted in the interest of brevity, but the total power (efficiency) computes to be 18%, again in agreement with Fujimoto, and the total ATU loss is 7.54 dB.

In Case 5, the resistance value is synthetic, for illustration purposes

| | |
|---|---|
| Initial $X_1$ - $X_3$ | +60, +60, 60 ohms |
| Optimized $X_1$ - $X_3$ | +60.507, +61.023, 59.708 ohms |
| Initial $Z_{incalc}$ | 50.501 + j0.099682 ohms |
| Optimized $Z_{incalc}$ | 50.000 + j3.6275e6 ohms |
| Final $R_1$ - $R_3$ | 0.60507, 0.18023, 0.059708 ohms |
| ATU input leg power | 1.2101 % |
| ATU shunt leg power | 0.20436 % |
| ATU output leg power | 0.24617 % |
| Power delivered to antenna | 98.33937 % |
| Total power | 100.000 % |
| Total ATU loss | 0.073 dB |

***Case 1* — Antenna $Z_{in}$ = 72 +$j$43 ohms, typical of a λ/2 dipole. Default $Q$ values for all cases were $Q_L$ = 100 and $Q_C$ = 1000.**

| | |
|---|---|
| Initial $X_1$ - $X_3$ | +31.62, +31.62, 31.62 ohms |
| Optimized $X_1$ - $X_3$ | +29.986, +32.254, 34.177 ohms |
| Initial $Z_{incalc}$ | 43.2 + j0.086 ohms |
| Optimized $Z_{incalc}$ | 50.0 j3.93e7 ohms |
| Final $R_1$ - $R_3$ | 0.299, 3.323, 0.034 ohms |
| ATU input leg power | 0.6 % |
| ATU shunt leg power | 0.2 % |
| ATU output leg power | 14.1 % |
| Power delivered to antenna | 85.1 % |
| Total power | 100.000 % |
| Total ATU loss | 0.70 dB |

***Case 2* — Antenna $Z_{in}$ = 20 – $j$300 ohms; a moderately mismatched small antenna.**

| | |
|---|---|
| Initial $X_1$ - $X_3$ | +24.19, +24.19, 24.19 ohms |
| Optimized $X_1$ - $X_3$ | +24.07, +24.34, 24.4 ohms |
| Initial $Z_{incalc}$ | 49.15 + j 0.098 ohms |
| Optimized $Z_{incalc}$ | 50.00 + j4.75e6 ohms |
| Final $R_1$ - $R_3$ | 0.241, 0.243, 0.024 ohms |
| ATU input leg power | 0.5 % |
| ATU shunt leg power | 0.3 % |
| ATU output leg power | 2.0 % |
| Power delivered to antenna | 97.2 % |
| Total power | 100.000 % |
| Total ATU loss | 0.12 dB |

***Case 3* — Antenna $Z_{in}$ = 11.7 + $j$0 ohms, a self resonant normal mode helical antenna (NMHA) of length 0.05λ, as described on page 68 of (4).**

| | |
|---|---|
| Initial $X_1$ - $X_3$ | +49.5, +49.5, 49.5 ohms |
| Optimized $X_1$ - $X_3$ | +0.12, +19.8, 24.5 ohms |
| Initial $Z_{incalc}$ | 2.62 + j5.13e3 ohms |
| Optimized $Z_{incalc}$ | 50.0 +j1.18e5 ohms |
| Final $R_1$ - $R_3$ | 0.0012, 9.198, 0.0245 ohms |
| ATU input leg power | 0.0 % |
| ATU shunt leg power | 0.3 % |
| ATU output leg power | 94.7 % |
| Power delivered to antenna | 5.0 % |
| Total power | 100.000 % |
| Total ATU loss | 12.97 dB |

***Case 4* — Antenna $Z_{in}$ = 0.49 – *j*900 ohms, for a short dipole reported by (5) and further considered in (4).**

| | |
|---|---|
| Initial $X_1$ - $X_3$ | +0.224, +0.224, 0.224 ohms |
| Optimized $X_1$ - $X_3$ | +64.6, +36.8, 28.8 ohms |
| Initial $Z_{incalc}$ | 0.007 + j9.1e6 ohms |
| Optimized $Z_{incalc}$ | 50.0 + j2.45e5 ohms |
| Final $R_1$ - $R_3$ | 0.65, 10.96, 0.03 ohms |
| ATU input leg power | 1.3 % |
| ATU shunt leg power | 0.5 % |
| ATU output leg power | 98.2 % |
| Power delivered to antenna | 9e3 % |
| Total power | 100.000 % |
| Total ATU loss | 40.5 dB |

***Case 5* — Antenna $Z_{in}$ = 0.001 + *j*11000 ohms. This is an extreme case of a 1 mH inductor being driven at 1.8 MHz through an ATU.**

only, and the radiation resistance is likely even smaller. Turns of the coil are necessarily tightly wound, resulting in high proximity effect losses, and the coupling of the coil to ground will also cause additional loss resistance in series at the feed terminals. An actual inductor approximating this case has been constructed using 285 turns of insulated #14 electrical wire, wound in a single layer on a nominal 4-inch diameter PVC pipe core. The measured inductance was 1.2 mH. In a rudimentary experiment, the inductive load did radiate at a level 50 to 60 dB down from a dipole, which proved sufficient to establish an interstate radio link under favorable noise and interference conditions.

The cited coil terminal resistance of 0.001 ohm is not a value realistically expected to be observed but, at the same time, is optimistic for a radiation resistance value in this case.

As expected, the ATU loss is enormous. For a real load device similar to that described, again, the observed input resistance would be much higher and the computed ATU loss in dB therefore lower. However, this would be a deceptive result because nearly all power delivered to the antenna terminals in that instance would be actually dissipated in ohmic loss versus radiation.

### Program Availability

Copies of the MATLAB code are available from the author on request by email. Please enter “ATU MATLAB code” in the email subject line. Please be advised that prospective users must have not only base MATLAB, but also the Optimization Toolbox, available to them.

### Concluding Remarks

Given accurate $Q_L$ and $Q_C$ values, the computer tool for ATU loss estimation described here has produced useful results in numerous test applications. Clearly, however, the reliability of the output depends directly on the precision of $Q$ specifications. It has proved challenging to discern more accurate “typical” $Q_L$ and $Q_C$ values for real components than those entered as the default numbers in the present code. Equipment is generally available for measuring $Q$ values and, because they are so important, ATU designers and users are urged to expend the time and effort necessary to obtain measured data in the context of their application and component implementations. Individuals willing to share their experiences, data, and/or conclusions about appropriate default inductor and capacitor $Q$ values are encouraged to contact the author.

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2. R.G. Brown, R.A. Sharpe, W.L. Hughes, and R.E. Post, *Lines, Waves, and Antennas – The Transmission of Electric Energy*, 2nd Edition, John Wiley & Sons, New York, 1973, pp. 160 and 112.

3. MATLAB® is a registered trademark of The Mathworks, Natick, MA.

4. K. Fujimoto, A. Henderson, K. Hirasawa, and J.R. James, J.R., *Small Antennas*, John Wiley & Sons, New York, 1987, p. 68.

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