Trigonometry and Circuits

Since we represent AC circuits as sine or cosine functions, and since Fourier expansion of more complex signals is simply a series of sine functions, it seems appropriate to review a few useful trig identities that can describe useful RF/microwave behaviors.

First, we remember the relationship between frequency f in cycles/sec (Hz) versus radians/sec (ω):

$$2\pi f = \omega$$

where ω is the variable commonly used for radians/sec.

Next, one cycle is 360º (one repetition that returns to the starting point), therefore:

$$2\pi \text{ radians} = 360^\circ$$

If we view the voltage of a sine wave on an oscilloscope, the sine function will tell us what the instantaneous voltage will be at any point on the waveform, relative to the peak voltage, since the maximum value of a sine function is 1.0. For example,

$$\sin(45^\circ) = 0.7071; \text{ voltage} = 0.7071\ V_{peak}$$

The final basic fact to remember is that the relationship between sine and cosine functions is simply $\pi/2$ radians, or 90º:

$$\cos(\alpha) = \sin(\alpha + \pi/2), \text{ or, } \cos A = \sin(A + 90^\circ)$$

Moving on to some trig function relationships, we find some useful expressions. Remember that sin (x) represents a signal at a specific frequency. The first example is a doubled angle (2× angular rate of change, or twice the frequency).

$$\sin 2\alpha = 2 \sin\alpha \cos\alpha$$

This means that a frequency doubler can be made by mixing (multiplying) a signal with a replica that is delayed by 90º. The above identity can be written in the form

$$2f_1 = 2[f_1 \times (f_1 + 90^\circ)]$$

We can use the following identity to describe the operation of a mixer:

$$\sin\alpha \sin\beta = 1/2 \cos(\alpha - \beta) - 1/2 \cos(\alpha + \beta)$$

The left hand side represents the tow input signals, while the right hand side shows the sum and difference frequencies that appear at the outputs.

We can extend this to two mixers, one of which has cosine inputs, that is, 90º delayed versions of the inputs to the other mixer. This is the classic I and Q scenario, where the second mixer operation is described as

$$\cos\alpha \cos\beta = 1/2 \cos(\alpha - \beta) + 1/2 \cos(\alpha + \beta)$$

Note that the right side of the equation is identical to the one with sine inputs, except for the sign between the two right hand constituents.. If we simply sum the outputs of these two mixers, we get

$$1/2 \cos(\alpha - \beta) + 1/2 \cos(\alpha - \beta) = \cos(\alpha - \beta)$$

This is another classic result of the I/Q mixing process, single-sideband, where only the difference product remains after summation. The sum product can be obtained either by taking the difference of the outputs, or by changing the sign of one of the mixer inputs.

Also note that, if the two mixer input signals are very close in frequency, the difference signal appearing at the output will be at a very low frequency. This is how a direct conversion receiver operates—the RF signal containing modulation sidebands is mixed with a local oscillator signal that has the same center frequency, but no modulation. Thus, after summation from the I and Q mixers, only the difference signal is present. The carrier, or center frequency, is mixed to zero, and only the sidebands of the input signal remain. A direct conversion transmitter simply reverses the direction of signal flow.

These simple expressions form the basis of more complex mathematical analysis of signals and their interactions. They can be expanded to include representations of series solutions and other common methods of analyzing the signals we work with every day.

We want to apologize for the incorrect author's name given in the January "Design Ideas" column. The correct name (Whitham, not "Whitman") is given below:

1. Whitham D. Reeve, *dc Power System Design for Telecommunications*, IEEE Press and John Wiley & Sons, Inc., 2007. ISBN 13-978-0-471-6816-1 and 10-0-471-68161-X, Ch. 4.