From June 2011 *High Frequency Electronics*
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Basic Electromagnetics Made True and Clear with Concrete Mathematics

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The author presents an alternate method of visualizing and analyzing the "point sources" required for electromagnetic field computations

The causes of electromagnetic phenomena are electric fields and magnetic fields; however, those fields are effects, as well as causes. The basic cause of an electric field, or a magnetic field, is one microscopic particle of electric charge and the way it behaves. One stationary particle of electric charge causes an electric field, and if that particle moves, it will also cause a magnetic field. Also, when its movement is time-varying, that causes two magnetic fields, and a second electric field.

The next-most-basic cause of electric fields and magnetic fields is a point source—a differential volume dV, in which there are numerous microscopic particles of electric charge. The net charge of all the charged particles in dV causes an electric field, and a net-charge time-variation, caused by particles moving in and out of dV, causes a second electric field. A net movement of all the charged particles in dV causes a magnetic field, and a time-variation of that movement causes a magnetic field and an electric field. A very common point source is any differential segment of an electric current of length $d\ell$, and cross section dA—a nanocurrent.

Charged Particle Fields

A stationary charged particle of q (coulombs) positioned at the origin of a spherical coordinate system, in an otherwise empty medium, has the r-directed electric field

$$E_r = \frac{Z_m v_p}{4\pi r^2} q \qquad \text{V/m}$$

where Z_m (ohms) is the medium's characteristic impedance; v_p (meters/second) is the field propagation velocity in the medium; and r (meters) is the distance from q to the observation point of E_r.

If q moves with a constant velocity of v (meters/second), in addition to E_r it causes the magnetic field

$$H_\phi = \frac{\sin\theta}{4\pi r^2} qv \qquad \text{A/m}$$

If q vibrates at $(r, \theta, \phi) = (0, 0, 0)$ with a velocity of $v(t) = v\sin(\omega t)$ (meters/second) parallel to the $\theta = 0$ axis, and $v \to 0$, then its time-varying movement adds to E_r the time-varying magnetic field

$$H_\phi(\tau) = \frac{\sin\theta}{4\pi r^2} qv(\tau) \qquad \text{A/m}$$

where $\tau = t - t_p$ was its cause time, and $t_p = r/v_p$ was its propagation time.

The time-variation of the charge movement $qv(t)$ (Fig. 1) also causes an electromagnetic field that consists of the magnetic field

$$\underline{H}_\phi(\tau) = t_p \frac{\partial H_\phi(\tau)}{\partial \tau} = t_p \left[\frac{\sin\theta}{4\pi r^2} q \frac{dv(\tau)}{d\tau} \right] \qquad \text{A/m}$$

and the electric field

$$\underline{E}_\theta(\tau) = Z_m \underline{H}_\phi(\tau) = Z_m t_p \left[\frac{\sin\theta}{4\pi r^2} q \frac{dv(\tau)}{d\tau} \right] \qquad \text{V/m}$$

Point Source Fields

A common point source is a designated differential segment dV, of length $d\ell$ and cross-

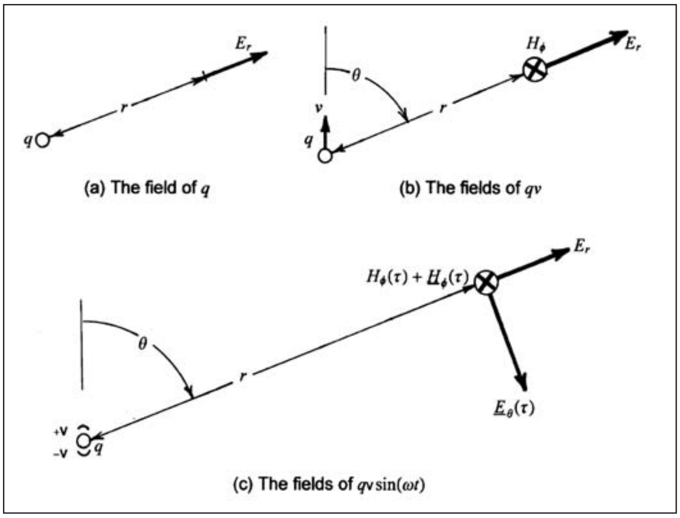

Figure 1 · Charged particle fields.

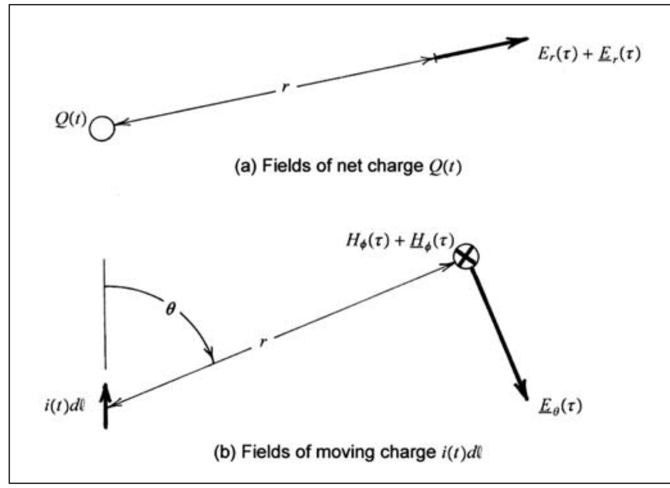

Figure 2 · Point-source fields of $Q(t)$ and $i(t)d\ell$.

section dA, of a conduction current $i(t)$. A difference in the currents entering and leaving dV, $i_{in}(t)$ and $i_{out}(t)$, causes a net charge in dV of

$$Q(t) = \int \left[i_{in}(t) - i_{out}(t) \right] dt \qquad \text{(C)}$$

—the net charge of all the charged particles in dV, both moving and stationary.

Therefore, the same as one charged particle, except for its time-variation, a net charge of $Q(t)$ in a point dV sends out the electric field

$$E_r(\tau) = \frac{Z_m v_p}{4\pi r^2} Q(\tau) \qquad \text{(V/m)}$$

And, the time-variation of $Q(t)$ adds to $E_r(\tau)$ the electric field

$$\underline{E}_r(\tau) = t_p \frac{\partial E_r(\tau)}{\partial \tau} = t_p \frac{Z_m v_p}{4\pi r^2} \frac{dQ(\tau)}{d\tau} \qquad \text{(V/m)}$$

If dV is centered at $(r, \theta, \phi) = (0, 0, 0)$, $d\ell$ is parallel to $\theta = 0$, and $i(t)$ is the average current in $d\ell$, then the charge movement in dV is $i(t)d\ell$ (coulombs/second × meters), and it causes the magnetic field

$$H_\phi(\tau) = \frac{\sin\theta}{4\pi r^2} i(\tau) d\ell \qquad \text{(A/m)}$$

The time-variation of the charge movement $i(t)d\ell$ (Fig. 2) causes an electromagnetic field, that consists of the magnetic field

$$\underline{H}_\phi(\tau) = t_p \frac{\partial H_\phi(\tau)}{\partial \tau} = t_p \frac{\sin\theta}{4\pi r^2} \frac{di(\tau)}{d\tau} d\ell \qquad \text{(A/m)}$$

and the electric field

$$\underline{E}_\theta(\tau) = Z_m \underline{H}_\phi(\tau) = Z_m t_p \frac{\sin\theta}{4\pi r^2} \frac{di(\tau)}{d\tau} d\ell \qquad \text{(V/m)}$$

A nanocurrent is a point source of moving charge and net charge. Its moving charge is $i(t)d\ell$, and its net charge is $Q(t) = i(t)d\ell/v_{pc}$ (coulombs), where $v_{pc} = d\ell/dt$ (meters/second) is the propagation velocity of two oppositely propagating electric fields that cause the current in dV. That $Q(t) = i(t)d\ell/v_{pc}$ is seen as follows.

Any conduction current is the charge flow rate

$$i(t) = \sigma E(t) dA = \frac{dq(t)}{dt} \qquad \text{(C/s)}$$

where σ (mhos/meter) is the conductivity in dV; $E(t) = E_{\curvearrowright}(t) + E_{\curvearrowleft}(t)$ is the electric field from the current source that forces movable charged particles to move; and $E_{\curvearrowright}(t)$ and $E_{\curvearrowleft}(t)$ are the oppositely propagating components of $E(t)$ from the current source terminals. Therefore,

$$i(t) = i_{\curvearrowright}(t) + i_{\curvearrowleft}(t) = \sigma E_{\curvearrowright}(t) dA + \sigma E_{\curvearrowleft}(t) dA \qquad \text{(C/s)}$$

which says that at each cross section dA of $dV = d\ell dA$, both $i(t)$ and $E(t)$ have two oppositely propagating components with velocities of v_{pc} and $-v_{pc}$ (meters/second).

Therefore, because $d\ell \to 0$, and $i_{in}(t)$ and $i_{out}(t)$ are its end-currents, the average current in $d\ell$, and the current at its midpoint, is

$$i(t) = \frac{i_{in}(t) + i_{out}(t)}{2} \qquad \text{(A)}$$

And, the net charge in dV is

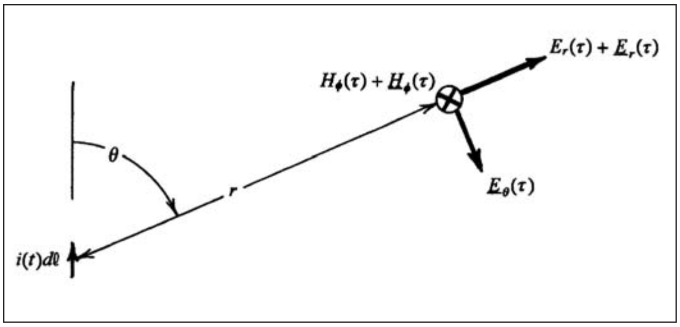

Figure 3 · The fields of a nanocurrent.

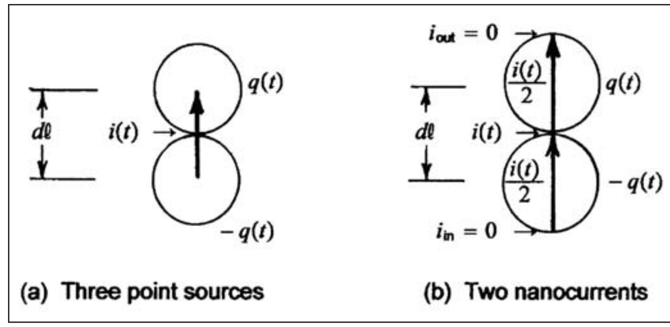

Figure 4 · Two viewings of a point dipole.

$$Q(t) = \int_0^t \left[i_{in}(t) - i_{out}(t)\right]dt \qquad \text{(C)}$$

Also, if $d\ell \rightarrow 0$,

$$\frac{i_{in}(t) - i_{out}(t)}{d\ell} = \frac{di(t)}{d\ell} \qquad \text{(A/m)}$$

and

$$\begin{aligned} Q(t) &= \int \left[i_{in}(t) - i_{out}(t)\right]dt = \int \left[d\ell \frac{di(t)}{d\ell}\right]dt \\ &= d\ell \int \frac{di(t)}{d\ell / dt} = \frac{d\ell}{v_{pc}} \int di(t) = \frac{i(t)d\ell}{v_{pc}} \end{aligned} \qquad \text{(C)}$$

where $dt = d\ell\, v_{pc}$ is the time that each component of $i(t) = i_{\curvearrowright}(t) + i_{\curvearrowleft}(t)$ takes to propagate through $d\ell$.

So, with $k = v_p / v_{pc}$, the fields of net charge of the nanocurrent $i(t)d\ell$ are

$$E_r(\tau) = \frac{Z_m k}{4\pi r^2} \frac{i(\tau)d\ell}{d\tau} = \frac{Z_m k}{4\pi r^2} i(\tau)d\ell \qquad \text{(V/m)}$$

and

$$\underline{E}_r(\tau) = t_p \frac{Z_m k}{4\pi r^2} \frac{di(\tau)}{d\tau} d\ell \qquad \text{(V/m)}$$

And, very often $v_{pc} \cong v_p$, so much of the time, for all practical purposes, $k = 1$.

The fields of the moving charge $i(t)d\ell$ are those previously given for a point source. Therefore, the fields of a nanocurrent are (Fig. 3)

$$\boldsymbol{E}_r(\tau) = E_r(\tau) + \underline{E}_r(\tau) = \frac{Z_m k}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

$$\boldsymbol{H}_\phi(\tau) = H_\phi(\tau) + \underline{H}_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

and

$$\boldsymbol{E}_\theta(\tau) = \underline{E}_\theta(\tau) = Z_m \underline{H}_\phi(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}\left[t_p \frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

Observations

The point dipole was conceived by Heinrich Hertz by viewing a current element in isolation, and assuming it has the current $i(t)$ from one end of $d\ell$ to the other. That would cause net charge accumulations on its ends of

$$+q(t) = \int_0^t i(t)dt \quad \text{and} \quad -q(t) = -\int_0^t i(t)dt \qquad \text{(C)}$$

However, as shown in Figure 4, the point dipole can be viewed as two nanocurrents. And, that says that the nanocurrent is a true current element, and the point dipole is a two-nanocurrent approximation of a current element.

The point-source fields $\boldsymbol{E} = E_r + \underline{E}_r + \underline{E}_\theta$, and $\boldsymbol{H} = H_\phi = \underline{H}_\phi$, can be *individually viewed* either as $\boldsymbol{E}(t)$ and $\boldsymbol{H}(t)$, or as $\boldsymbol{E}(\tau)$ and $\boldsymbol{H}(\tau)$. However, when $\boldsymbol{E}$ and $\boldsymbol{H}$ are related to each other, as in Maxwell's equations

$$\nabla \times \boldsymbol{E}(t) = -\frac{Z_m}{v_p}\frac{\partial \boldsymbol{H}(t)}{\partial t} \quad \text{and} \quad \nabla \times \boldsymbol{H}(t) = \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}(t)}{\partial t}$$

the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ must be viewed as functions of $\tau = t - r/v_p$. In those equations, $\boldsymbol{E}$ and $\boldsymbol{H}$ cannot be viewed as functions of t, because there are r-derivatives in $\nabla \times \boldsymbol{E}$ and $\nabla \times \boldsymbol{H}$, and

$$\frac{\partial i(t)}{\partial r} = 0 \quad \text{but} \quad \frac{\partial i(t)}{\partial r} = \frac{\partial i\left(t - r/v_p\right)}{\partial r} = -\frac{1}{v_p}\frac{\partial i\left(t - r/v_p\right)}{\partial t}$$

Also, because the nanocurrent fields $E_r(\tau)$ and $\underline{E}_r(\tau)$ are *not* functions of θ and the nanocurrent fields $\underline{E}_\theta(\tau)$, $H_\phi(\tau)$ and $\underline{H}_\phi(\tau)$ are functions of θ, it follows that

$$\nabla \times \boldsymbol{E}(\tau) \neq -\frac{Z_m}{v_p}\frac{\partial \boldsymbol{H}(\tau)}{\partial t} \quad \text{and} \quad \nabla \times \boldsymbol{H}(\tau) \neq \frac{1}{Z_m v_p}\frac{\partial \boldsymbol{E}(\tau)}{\partial t}$$

Thus, the fields of the nanocurrent—a true current element—do not satisfy James Maxwell's point-source field equations.

Conclusions

As noted by at least one author more than three decades ago, "Current elements have no separate existence" [1]. Therefore, for use as a current element, the point dipole should be replaced with the nanocurrent—a true current element (Fig. 5).

And, based on the above observations, Maxwell's equations should be carefully re-examined, and the overuse of abstract mathematics that they apparently have caused should be replaced with the use of concrete mathematics.

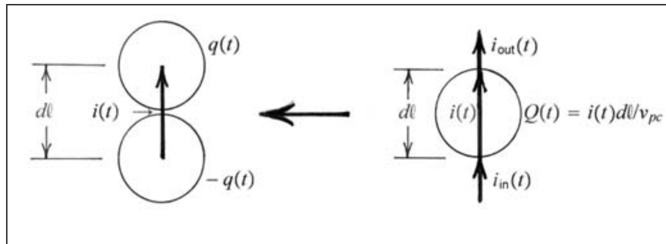

Figure 5 · A needed replacement.

Reference

1. Joseph A. Edminister, *Theory and Problems of Electromagnetics*, McGraw-Hill, 1979, page 113.

Author Information

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Editor's Note

From time-to-time we publish works such as this one, which can be characterized as "exploratory" or "speculative." Although Dr. Bennett is challenging Maxwell's equations, his intent is to find a better explanation and definition of an important aspect of those equations. Feedback and critique of his proposed analysis is welcome.

—Gary Breed, Editorial Director