

# The Twisted-Pair Telephone Transmission Line

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This article reviews the principles of operation and measurement methods for twisted pair (balanced) transmission lines commonly used for xDSL and ethernet computer networking

**T**elephone line is a balanced twisted-pair transmission line, and like any electromagnetic transmission line, its characteristic impedance  $Z_0$  can be calculated from manufacturers' data and measured

on an instrument such as the Agilent 4395A (formerly Hewlett-Packard HP4395A) network analyzer. For lowest bit-error-rate (BER), central office and customer premise equipment should have analog front-end circuitry that matches the telephone line impedance. This article contains a brief mathematical derivation and a computer program to generate a graph of characteristic impedance as a function of frequency.

Twisted-pair line for telephone and LAN applications is typically fashioned from #24 AWG or #26 AWG stranded copper wire and will be in one of several "categories." The Electronic Industries Association (EIA) and the Telecommunications Industry Association (TIA) define some North American standards for unshielded twisted-pair cables and classify them into categories based upon deployment requirements. EIA/TIA-568A, for example, is a commercial building telecommunications wiring standard.

- Category 1:  $BW_{MAX} < 1$  MHz. No performance criteria. Good only for analog voice communication, RS232, RS422 and ISDN, but not for data. Used in POTS (Plain Old Telephone System).
- Category 2:  $BW_{MAX} < 1$  MHz. Used in telephone wiring.

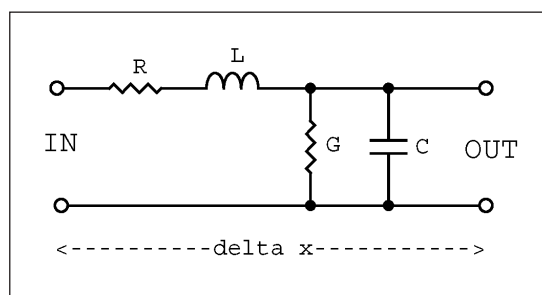


Figure 1. Lumped element model of a transmission line.

- Category 3:  $BW_{MAX} < 16$  MHz. Intended for older networks and telephone systems in which performance over frequency is not especially important. Used for voice, digital voice, older ethernet 10Base-T and commercial customer premise wiring. The market currently favors CAT5 installations instead.
- Category 4:  $BW_{MAX} < 20$  MHz. Not much used. Similar to CAT5 with only one-fifth the bandwidth.
- Category 5:  $BW_{MAX} < 100$  MHz. Have tightly twisted pairs for low crosstalk. Used for fast ethernet 100Base-T, 10Base-T. #22AWG or #24 AWG wire pairs.
- Category 6:  $BW_{MAX} < 350$  MHz. Under development.
- Category 7:  $BW_{MAX} < 600$  MHz. Under development.

## A Bit of Derivation

Recall a transmission line model composed of discrete resistors, capacitors and inductors. A length  $x$  of transmission line can conceptually be divided into a large (infinite) number of increments of length  $\Delta x$  ( $dx$ ) such that the

series and shunt  $R_s$ ,  $L_s$  and  $C_s$  are given as shown in Figure 1.

$G (=1/R_{shunt})$  could have been given a resistor “ $R_{shunt}$ ” label, but analytically it is often simpler to deal with reciprocal resistance values for shunt resistors. There are many old and new textbooks on electricity and magnetism in which transmission line derivations can be found [for example, References 6, 7, 8 and 9]. A real twisted pair telephone line has resistance, capacitance and inductance distributed equally between both wire members of the pair as shown above (Figure 2) in a segment of line approximated by three segments of length  $\Delta x$ . For mathematical modeling, however, the single-sided model of Figure 1 is easier to comprehend and analyze.

An ideal transmission line would be lossless; that is,  $R = 0$  and  $G = 0$  (i.e.,  $R_{shunt} = \infty$ ), as represented in Figure 3. For such a line, the characteristic impedance is given by the familiar formula

$$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{L}{C}} \tag{1}$$

However, for a real transmission line, the characteristic impedance is

$$Z_0 = \sqrt{\frac{R' + i\omega L'}{G' + i\omega C'}} \tag{2}$$

where

$$R' = \frac{dR}{dx}, \quad L' = \frac{dL}{dx}, \quad C' = \frac{dC}{dx}, \quad G' = \frac{dG}{dx}$$

are the longitudinal derivatives of resistance, inductance, capacitance, and admittance along the line. Keep in mind that  $Z_0$  is a complex valued function. The values of these parameters and their tolerances can be found tabulated in the catalogs of cable manufacturers, usually under the designation “RLCG parameters.” Some engineers make impedance magnitude measurements at 1 kHz and are puzzled why their measured values do not agree with manufacturers’ data. Each of the parameters  $R'$ ,  $L'$ ,  $C'$ , and  $G'$  is frequency dependent. For example,  $R'$  will change in value due to skin effect, and  $G'$  will change in

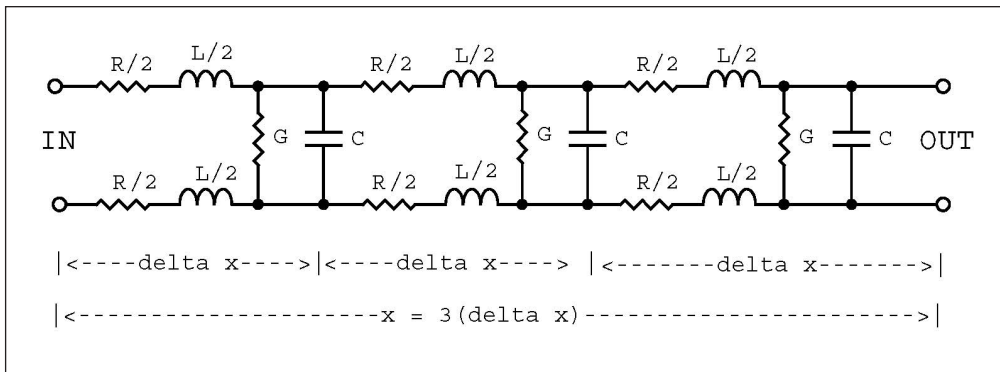


Figure 2 · A balanced line model in three increments.

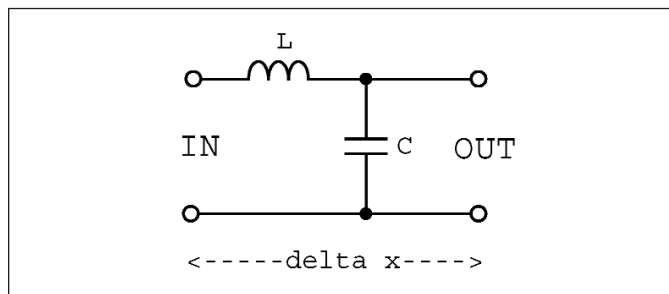


Figure 3 · Lumped element model of an ideal lossless line.

value due to frequency dependent dielectric loss. Consequently,  $|Z_0|$  measured at one frequency will not necessarily be the same as  $|Z_0|$  measured at another frequency. This deviation is especially notable at the audio frequencies (up to 100 kHz). As a rule of thumb, it is advisable to conduct characteristic impedance measurements at frequencies greater than or equal to 100 kHz, where  $|Z_0|$  begins to approach an asymptotic value. From Equation 2, we can calculate the complex valued  $Z_0$ . Introduce  $u$  as a temporary additional variable to avoid clutter and confusion in the derivation. Then,

$$u = Z_0^2 = \frac{R' + i\omega L'}{G' + i\omega C'} = \frac{(R' + i\omega L')(G' + i\omega C')}{G'^2 + \omega^2 C'^2} \tag{3}$$

$$= \frac{(R'G' + \omega^2 L'C') + i\omega(L'G' - R'C')}{G'^2 + \omega^2 C'^2}$$

$$uu^* = |u|^2 = \frac{(R'G' + \omega^2 L'C')^2 + \omega^2 (L'G' - R'C')^2}{(G'^2 + \omega^2 C'^2)^2} \tag{4}$$

$$\therefore |u| = \frac{\sqrt{(R'G' + \omega^2 L'C')^2 + \omega^2 (L'G' - R'C')^2}}{G'^2 + \omega^2 C'^2} \quad (5)$$

$$|Z_0| = \sqrt{|u|} = \frac{\sqrt[4]{(R'G' + \omega^2 L'C')^2 + \omega^2 (L'G' - R'C')^2}}{\sqrt{G'^2 + \omega^2 C'^2}} \quad (6)$$

We now can use Equation 6 to calculate the *magnitude* of  $Z_0$ . Remember that  $Z_0$  is a *complex* quantity,

$$Z_0 = R_0 + iX_0 \quad (7)$$

where  $R_0$  is the real part (resistance) of  $Z_0$  and  $X_0$  is the imaginary part (reactance) of  $Z_0$ . If  $X_0 > 0$ , then the reactance is inductive, and if  $X_0 < 0$ , then the reactance is capacitive.

### Frequency Dependence of Transmission Line Parameters

There are a number of continuous equations that can be used to express  $R'$ ,  $L'$ ,  $C'$ , and  $G'$  as functions of frequency. Here, we use some equations and parameter values found in Reference [4] and consider #24 AWG twisted pair copper wire.

$$R'(f) = \frac{1}{\frac{1}{\sqrt[4]{R_{0c}^4 + a_c f^2}} + \frac{1}{\sqrt[4]{R_{0s}^4 + a_s f^2}}} \quad (8)$$

$$L'(f) = \frac{L'_0 + L'_\infty \left(\frac{f}{fm}\right)^b}{1 + \left(\frac{f}{fm}\right)^b} \quad (9)$$

$$C'(f) = C'_\infty + C'_0 f^{-c_c} \quad (10)$$

$$G'(f) = G'_0 f^{g_c} \quad (11)$$

Similar equations can be found in Reference [3], [5] and in a number of other sources.

In the above equations we use primes to express derivatives. More often than not, in texts on electromagnetic theory, transmission lines and in other literature (and in References [3] and [4]), the primes are omitted and the derivative sense is understood.

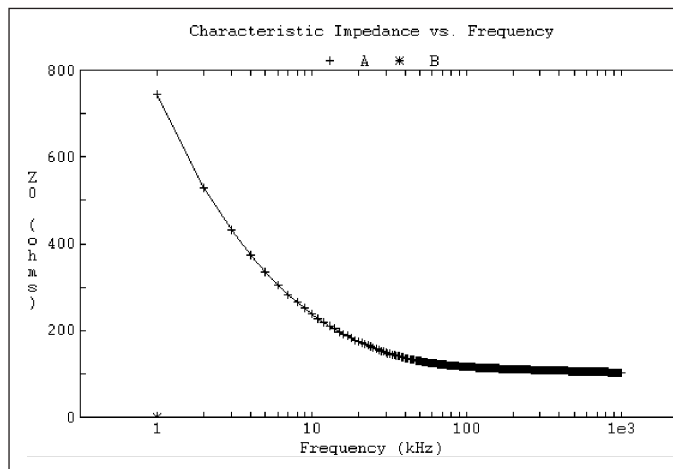


Figure 4 - A plot of characteristic impedance versus frequency.

### Computer Calculations

A computer program (listing in Table 1), written in TrueBASIC, calculates and tabulates values of  $|Z_0|$  and plots them as a function of frequency. As frequency gets larger and larger, the value of  $|Z_0|$  approaches an asymptotic value. The source code is also useful as pseudo-code for writing a  $Z_0$  program in any programming language. This impedance is that of a real transmission line; that is, the transmission line has electrical leakage characterized by the conductance per unit length,  $G'$ . The program performs these functions:

- Use Equations 8 through 11 to compute  $R'$ ,  $L'$ ,  $C'$  and  $G'$  at specific frequencies.
- Substitute these values into Equation 6 to obtain the magnitude of the characteristic impedance for various frequencies.
- Plot the magnitude of cable characteristic impedance as a function of frequency (as in Figure 4).

### Measuring Characteristic Impedance $Z_0$

Use an impedance measuring instrument such as a Hewlett-Packard (Agilent) HP4395A network analyzer and HP43961A impedance test adapter. Manufacturer's specification guarantees impedance measurement accuracy down to 100 kilohertz; however, lower frequencies can be included with reduced accuracy, especially to compare data with that in Figure 4 or to focus attention on the lower frequency range occupied by ADSL signals. For example, we set the lower frequency to 10 kilohertz to 30 kilohertz in our laboratory. For the purposes of this paper, however, set up the instrument to measure magnitude of impedance from 100 kilohertz to 1 megahertz. Measure open-circuit impedance,  $Z$  (with the other end of the line open; that is, not connected to anything). Then measure

TRANSMISSION LINES

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! The name of this TrueBASIC program is Z0.TRU.
! The purpose of this program is to compute the characteristic impedance, Z0,
! of a twisted pair wire line as a function of frequency. The resistance per
! unit length, capacitance per unit length, and inductance per unit length
! as a function of frequency are first computed.
! The values provided below in the initialization of this program
! are found in Reference 4.
! r = dR/dx, l = dL/dx, c = dC/dx, g = dG/dx, etc.
! #24 AWG twisted pair copper wire for telecommunications.

LIBRARY "D:\TB Gold\tblibs\sglib.trc"

DIM x(2,1000), y(2,1000), legends$(2)
MAT READ legends$
DATA A,B
option nolet

r0c = 174.55888 ! Ohms, resistance/km.
r0s = 1E20 ! infinity
ac = 0.053073481
as = 0
5
l0 = 617.29539E-6 ! Henries, inductance/km.
linf = 478.97099E-6 ! Henries, inductance/km.
cinf = 50E-9 ! Farads, capacitance/km.
c0 = 0
ce = 0
g0 = 234.87476E-15
fm = 553.760E3 ! Hertz
b = 1.1529766
ge = 1.38

For k = 1 to 1000
f = 1000*k ! 1 kz to 1 MHz.
  omega = 2*PI*f
  term1 = r0c^4 + ac*f*f
  term2 = r0s^4 + as*f*f
  r = 1/(1/(term1^(1/4)) + 1/(term2^(1/4))) ! That is, R' in ohms/km.
  l = (l0 + linf*(f/fm)^b)/(1 + (f/fm)^b) ! That is, L' in henries/km.
  c = cinf + c0*f^(-ce) ! That is, C' in farads/km.
  g = g0*f^ge ! That is, G' in siemens/km.
  ! Calculations:
  term3 = (r*g + omega*omega*l*c)^2 + omega*omega*(l*g - r*c)^2
  Z0 = (term3^(1/4))/(sqr(g*g + omega*omega*c*c)) ! That is, |Z0|.
  x(1,k) = k
  y(1,k) = Z0
  x(2,k) = 1 ! Not used here.
  y(2,k) = 1 ! Not used here.
  print "Characteristic impedance, Z0 =";Round(Z0,1);" ohms @";f/1000;" kHz."
next k
CALL SetText("Characteristic Impedance vs. Frequency", "Frequency (kHz)",
"Z0 (ohms)")
CALL SetGraphType("logx")
colors$ = "red green magenta cyan"
CALL ManyDataGraph(x,y,1,legends$, color$)
End

```

the short-circuit impedance  $Z'$  (with the other end of the line short-circuited). The characteristic impedance of the line,  $Z_0$ , is

$$Z_0 = \sqrt{ZZ'} \quad (12)$$

$R'$ ,  $L'$ ,  $C'$ , and  $G'$  all vary with frequency. Hence,  $Z_0$  also varies with frequency. However,  $Z_0(f)$  monotonically decreases and approaches an asymptote,  $Z_0$ . If only one or two frequencies are available for measurement, choose frequencies greater than or equal to 100 kHz.

Consider the following twisted-pair cable:

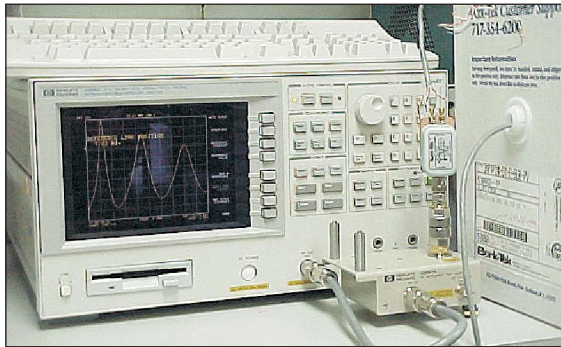
Berk-Tek telephone cable, part number 530462-TP, Category 3, PVC, non-plenum, # 24AWG, 4 unshielded twisted-pairs (UTP), Nominal characteristic impedance,  $|Z_0| = 100$  ohms  $\pm 15$  ohms. Cable length = 1000 feet = 305 meters; Intended for LAN and medium speed data up to 16 Mbps (Reference 2). Speed of propagation of EM wave,  $v = 70\%$   $c$ .

Figure 7 is a plot of the impedance magnitude measured on this cable.

This cable had the virtue of being immediately available in our laboratory for measurements. The HP43961A RF impedance test adapter was positioned on the front of the HP4395A network analyzer, and a North Hills 0001BB wideband transformer was then mounted on top of the HP43961A. The wideband transformer has a 1:1 turns ratio with a single-ended primary winding and balanced secondary winding.

The balun transformer is necessary, because we wish to measure differential impedance, not common-mode impedance. It is differential impedance that cable manufacturers specify in their data sheets. The wire was left rolled up in its shipping box, one end of one wire pair (green and white) was connected to pins 3 and 4 (the balanced secondary winding) of the North Hills transformer, and the other end of the twisted pair was left either open or short-circuited; that

Table 1 · TrueBASIC program listing for computing and plotting  $Z_0$ .



Instruments and modules required:  
 HP4395A Network/Spectrum/Impedance Analyzer.  
 HP43961A RF Impedance Test Adapter.  
 North Hills 0001BB Wideband Balun Transformer or equivalent. (100 kHz – 125 MHz).

Figure 5 · Measurement setup for  $Z_0$ .

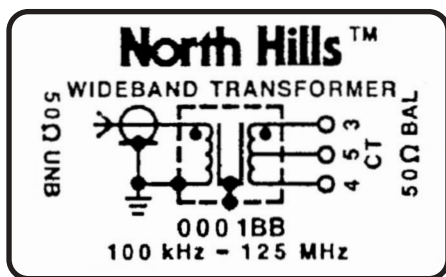


Figure 6 · North Hills 0001BB Balun Transformer.

is, the cable was not terminated in its characteristic impedance. The impedance curves for each of these situations cross one-another regularly, so that it is convenient to measure  $|Z_0|$  and these points, because both  $Z$  and  $Z'$  (see Equation 12) will have the same value there. The impedance magnitude was between approximately 101 and 106 ohms for frequencies between 258 kHz and 905 kHz.

[Note: It is a useful exercise to also run these measurements without the balun transformer to measure the common-mode impedance. The two impedance values will probably be within 10% of one another.]

### Fluctuations in the Graph

But what of the periodic fluctuations in the above graphs? Why are there fluctuations in the graph of Figure 7 obtained from measurements, but not in the graph of Figure 4 obtained from theory? Let us rehash a few basic concepts from transmission line theory. In ordinary mea-

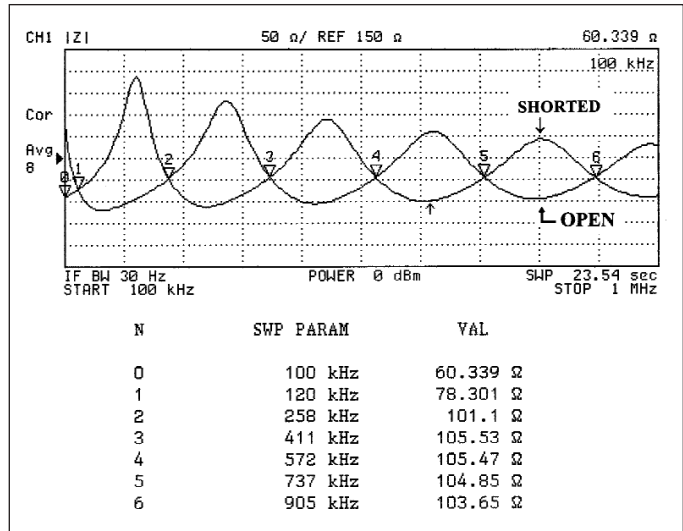


Figure 7 · Short-circuit and open-circuit cable measured impedance.

surements on the bench top of circuits at DC and audio frequencies we implicitly assume that all of the power delivered to a load is absorbed by the load, because the signal lines are usually a very small fraction of a wavelength long. However, in telephone lines this is not so; telephone lines are twisted-pair cables that can be many kilometers long. The graph of Figure 4 represents measurements of a cable 0.305 km long.

From classical electromagnetic theory, Load reflection coefficient,  $\rho_L$  equals the reflected wave voltage amplitude at termination  $R_L$  divided by the incident wave voltage amplitude at termination  $R_L$ , or

$$\rho_L = \frac{R_L - Z_0}{R_L + Z_0} \quad (13)$$

- For open-circuit load,  $R_L = \infty$ , and  $\rho_L = +1$ . (Total reflection).
- For short-circuit load,  $R_L = 0$ , and  $\rho_L = -1$ . (Total reflection and inversion).
- For matched load impedance,  $R_L = Z_0$ , and  $\rho_L = 0$ ; that is, there is no reflected wave.
- Hence, the S-parameter measurement condition (zero voltage for reflected wave) is satisfied if  $R_L = Z_0$ .

The fluctuations in the graph indicate the presence of a standing wave on the cable generated by reflection of the signal from a fault. And what is the nature of the fault? The fault is due to terminating the cable in zero ohms impedance (short-circuit) or infinite ohms (open-circuit) rather than in its proper termination, an impedance equal to the characteristic impedance of the line. In most



audio range electronic circuits we also encounter a partial reflection of signal from the load (unless the load is matched), and do not give it a second thought nor worry about it, because the amplitude of the reflected signal is negligible. However, when we are designing communications circuits for xDSL applications (hybrids in particular), these small amplitude reflected signals assume importance.

**A Variation on Time Domain Reflectometry (TDR)**

Time Domain Reflectometry is the analysis of a transmission line by sending a voltage pulse down the line, and then examining the reflection of that pulse. The signature of the return pulse (polarity, amplitude, shape, delay time) carries information about the nature of the termination. Frequently a termination will be a “fault,” an unintentional and undesired impedance discontinuity due to a break, a leak, or short-circuit in the cable. In such a situation, TDR is used to determine the location of the fault. In geophysics TDR is used to determine soil moisture, conductivity and rock mass deformation. Reflectometry involves calculations based upon the speed of electro-magnetic waves and the dependence of this speed upon physical properties of the transmitting medium; e.g., resistance, capacitance and inductance.

However, the same principles apply whether a pulsed voltage signal is used or a continuous voltage signal is used. Using the Agilent 4395A, we depend upon continuous waves to make measurements.

**Derivation**

In general, for an EM wave propagating down a transmission line and being reflected,

$$2d = n\lambda_1 = (n+1)\lambda_2 \tag{14}$$

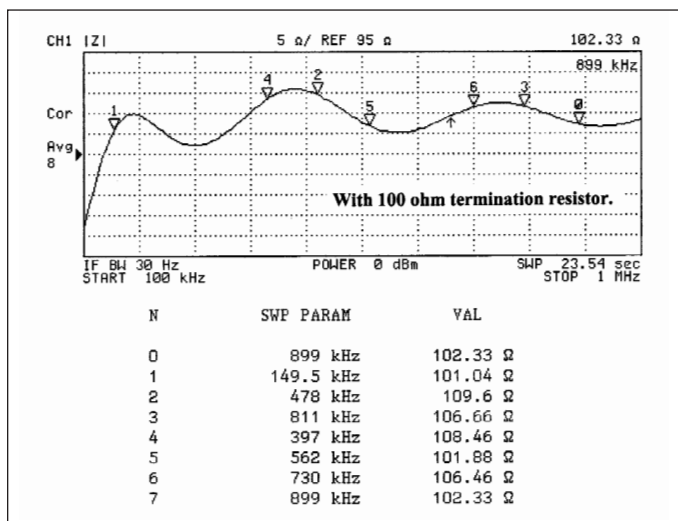


Figure 8 . Properly terminated cable measured impedance.

where  $d$  is the distance from the signal source (the North Hills balun transformer) to the other end of the cable and  $n$  is an integer. The factor of 2 occurs, because the wave travels a distance  $d$  from source to the end of the cable, is reflected there, and travels that same distance  $d$  back to the source. In general,

$$v = f\lambda \tag{15}$$

$$v_1 = f_1\lambda_1, v_2 = f_2\lambda_2, v = v_1 = v_2 \tag{16}$$

where  $f$  is frequency,  $\lambda$  is wavelength and  $v$  is velocity of wave propagation. Then, solve for the difference in frequencies (That seems to be the quickest way through the derivation)...

$$\begin{aligned} \Delta f &= f_2 - f_1 = \frac{v}{\lambda_2} - \frac{v}{\lambda_1} = v \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) \\ &= v \left( \frac{n+1}{2d} - \frac{n}{2d} \right) = \frac{v}{2d} \end{aligned}$$

$$\text{Therefore } v = 2d(f_2 - f_1) = 2d\Delta f \tag{17}$$

End of derivation.

**Returning to Our Problem**

But what if we terminate the far end of the cable in, say, a resistor equal to 100 ohms, the approximate value of the cable's characteristic impedance?

The length of the cable is  $x = 1000$  feet = 305 meters. The Figure 7 tic marks 4 and 6 are positioned at frequencies  $f_4 = 572$  kHz and  $f_6 = 905$  kHz respectively.  $f = f_6 - f_5 = 333$  kHz for this particular cable. Let the speed of wave propagation be  $v$ , and assume that it is the same for waves of both frequencies. The wave travels from the generator to the impedance discontinuity and back to the generator and this distance must be an integral number of wavelengths.

Using Equation 17,  $v = (2d)(\Delta f) = 2d(f_6 - f_5) = 2(305 \text{ meters})(333 \times 10^3 \text{ Hz}) = 2.0313 \times 10^8 \text{ m/sec}$ . The speed of EM propagation (“speed of light”) in free space is  $c = 3 \times 10^8 \text{ m/sec}$ . Therefore, the ratio is  $\beta_1 = v/c = (2.0313 \times 10^8 \text{ m/sec}) / (3 \times 10^8 \text{ m/sec}) = 0.677$  (dimensionless); that is,  $v = 67.7\% c$ . This figure agrees fairly well with that provided by the manufacturer, 70%.

The nominal impedance of the cable provided by the manufacturer is 100 ohms. In Figure 7, the actual impedance of the cable sample was approximately 105.5 ohms at 0.5 MHz, slightly different from the nominal value, but well within tolerance. By way of contrast to Figure 7, we see in Figure 8 that when a 100 ohm resistor (value of the nominal impedance) is connected across the

wires at the far end, the cable is almost properly terminated and that the fluctuations in the  $|Z_0|$  curve are much smaller. However, even if we terminate the cable with a resistor equal to 105.5 ohms, the small fluctuations in the impedance-versus-frequency graph will persist.

This is to be expected, for the following reasons:

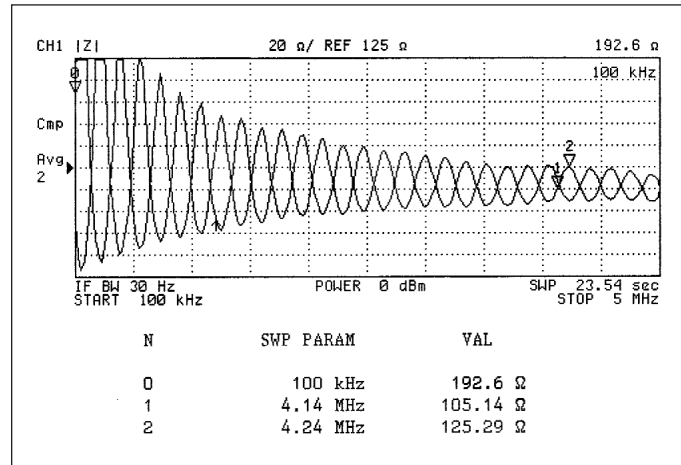
Recall from Equation 7 that in a real transmission line (the telephone cable twisted-pair) the impedance has resistive and reactive components and that the reactive component gets smaller and smaller as the frequency is increased. Nevertheless, the reactive component is still present at the frequencies of measurement. The characteristic impedance is  $Z_0(f) = R_0(f) + iX_0(f)$ , and we are attempting to match this impedance  $Z_0$  with a resistor equal to  $|Z_0|$  even though  $X_0(f) \neq 0$ . This attempt is bound to fail. Figure 9 is an impedance-frequency plot similar to that of Figure 7, but stretching from 100 kHz to 5 MHz. The separation between maxima and minima is successively smaller with increasing frequency.

The unshielded twisted pair (UTP) cable was coiled in a box, with each turn of wire resting on adjacent turns of wire. Notwithstanding that we were measuring differential impedance, the distance between wires of adjacent loops was comparable to the wire-to-wire distance of the pair in the same loop. Hence, the capacitive and inductive coupling between loops is somewhat significant. It would be better, although perhaps troublesome, to unroll the cable from the reel for these measurements.

The tic marks for points (4.14 MHz, 105.14 ohms) and (4.24 MHz, 125.29 ohms) are provided merely to give an indication of scale on the graph.

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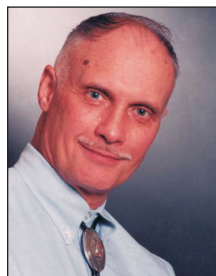


**Figure 9 · Short-circuit and open-circuit cable measured impedance over 5 MHz.**

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