

Key Relationships Between Time and Frequency Domains

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The observation and analysis of signals in both time domain and frequency domain is especially important today, given recent developments in digital signal processing and complex modulation

With today's complex modulation formats, digital IF and baseband signal processing, frequency synthesis and control systems, the relationship between the time domain and frequency domain has become an essential

concept for all engineers to understand. This tutorial will present some of the basic relationships between these two perspectives. The emphasis is on identifying what factors are important and their effects.

Signals in Time and Frequency

Events observed only in time are sequential (e.g. the rise and fall of a sine wave), while in the frequency domain, we are looking at cyclical events (e.g. the same sine wave appears as a single frequency). In other words, to observe the signal in the time domain, you use an oscilloscope. For frequency domain observations, you use a spectrum analyzer.

Table 1 lists some of the most common time domain measurements, observations and analyses of RF signals, while Table 2 lists the common observations of digital signals in the frequency domain. These represent the most basic elements that "crossover" between the two observational domains, where the opposite perspective is used to evaluate the behavior in the normal domain.

As we all learned in our undergraduate Linear Systems class, there are much more detailed mathematical relationships between the time and frequency domains. The Fourier transform is the foundation of time/frequency

- Oscillator startup behavior
- Clipping or limiting of signals
- Waveform distortion (usually to determine cause of harmonics or spurs)
- Analysis and measurement of modulation envelope properties
- On/off switching transition behavior
- Transients (DC power shifts, effects from other portions of the circuit)
- Power supply regulation effects
- Noise analysis

Table 1 · Some common time domain observations of RF signals.

- Electromagnetic compatibility (EMC) measurements and analysis
- Signal integrity analysis (verify frequency range of Fourier coefficients/harmonics)
- Analyze jitter as phase noise
- Verify performance of digital filters and other digital signal processing
- Analyze performance of digital-to-analog converters (DACs) in direct digital frequency synthesis (DDS) and other waveform generation applications

Table 2 · Some common frequency domain observations of digital signals.

mathematics, with the basic form:

$$F(y) = \int_{-\infty}^{\infty} f(x) \exp(-j2\pi xy) dx$$

where $F(y)$ is the frequency domain representation of the time domain function, $f(x)$.

The above equation is for an impulse func-

tion; a single event in the time domain. While impulse response is important, we are more often interested in periodic waveforms (signals that are present in the frequency domain), for which the Fourier transform takes the form of a series:

$$f(x) = \frac{1}{2} A_0 + A_1 \cos x + A_2 \cos 2x + A_3 \cos 3x + \dots$$

$$\dots + B_1 \sin x + B_2 \sin 2x + B_3 \sin 3x + \dots$$

which can be reduced to:

$$f(x) = \frac{1}{2} A_0 + \sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx)$$

where,

$f(x)$ is a periodic function with a period $2p$, that is, its values are completely defined by an interval from 0 to $2p$ radians (or $-p/2$ to $+p/2$).

A_0 is the initial value (analogous to a DC offset).

A_n, B_n are the coefficients of each term of the series

The Fourier series equation gives us the familiar representation of a time domain function as a collection of fundamental and harmonic frequencies. Although the Fourier series is infinite, for many signals, sufficient accuracy can be obtained by using a finite number of terms. At some point, the coefficients (A_n, B_n) are small enough to be ignored. However, there are various waveforms that require a large number of terms before the coefficient values reach small enough values.

One such waveform is a repeating pattern of very short pulses (Figure 1a), where each pulse has a duration of t and the repetition rate is τ . In this case, the coefficients will occur every $1/\tau$ in frequency, with amplitudes that fit into an “envelope” defined by a $(\sin x)/x$ function. The first zero crossing will be at a frequency of $1/t$, designated f_1 in Figure 1b.

If the pulse is very short ($t \ll \tau$), the number of discrete frequency terms between zero and f_1 will be very large. Several zero crossing periods are required before the amplitude of the coefficients is small enough to ignore. This illustrates that a large number of Fourier series terms may be required to accurately represent a relatively simple waveform.

Without exploring the mathematics any further, it should be apparent that digital signals require analysis in the frequency domain as well as the more familiar time domain. We’ll review a few areas where this analysis is of current interest.

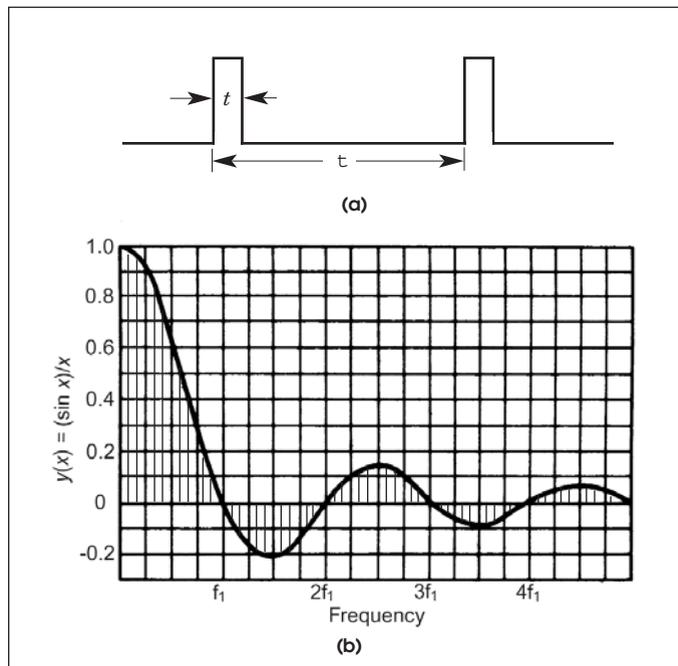


Figure 1 · Time domain (a) and frequency domain (b) representations of a waveform comprising a series of short pulses.

Electromagnetic Compatibility (EMC)

EMC is a well-established engineering specialty for the measurement, troubleshooting and prevention of unwanted radiation from electronic equipment. Digital equipment is particularly important, because the pulse train of bits has significant energy at frequencies that are several multiples of the clock rate, as illustrated in the previous discussion.

In the past, when digital equipment was just beginning to become commonplace, clock rates were in the tens of MHz. EMC concerns were limited to frequencies below 100 MHz. Remedies for excessive radiation were relatively easy, using bypass capacitors, shielding, ferrite absorbing materials and good layout techniques.

Now, with clock rates into the GHz region, significant energy is present well into the microwave frequency range. At higher frequencies, the energy is more difficult to contain. Subtle factors like slots in an enclosure, length of individual component interconnections and radiation directly from (supposedly) terminated p.c. board traces all are potential causes for unwanted radiation.

Digital Signal Integrity

These same high frequency factors affect the accuracy of the actual pulse train of digital signals as they are conducted from component-to-component, or within components themselves. These frequency-domain issues that affect digital signals have not been familiar to digital

engineers until relatively recently. Some of the major factors are:

- Reflected waves from circuit board traces that are not designed as transmission lines and/or not properly terminated.
- Reflected waves from discontinuities—bends in p.c. board traces, thickness changes due to components and solder, integrated circuit pads and internal connections, and gaps in the ground plane layer.
- Reduced signal voltage amplitude due to dielectric losses, radiation losses and impedance mismatches.
- Unequal propagation delays in parallel busses.
- Timing errors.
- Crosstalk between signal lines and other types of self-interference.

These issues have been widely discussed among the high-level digital engineers involved in high speed computing, communications and digital imaging. However, the information is less well-known than it should be to many engineers. In some cases, the old saying that digital engineers don't understand RF has been turned around—RF engineers incorporating high-speed digital components in their wireless products may not understand the full range of digital issues.

Time Domain Analysis of RF Signals

The greatest need for information may be in the high-speed digital realm, but there are significant areas where time domain techniques are essential to understanding current RF/microwave communications.

One of these areas is reducing distortion to meet emissions mask standards. Feedback, feedforward and predistortion techniques all require accurate timing, and may include adaptive capabilities that require signal sampling, analog-to-digital conversion, computation and digital-to-analog conversion. Accuracy and timing in such systems is critical.

Another interesting area requiring both time and frequency analysis is propagation analysis. The time delays and amplitude variations in multipath signals, particularly in a mobile environment, create a number of errors and distortions that require a combination of time and frequency techniques to identify and correct. Engineers working in this area will be familiar with histograms, waterfall displays and other methods that capture frequency data over time.

Finally, new techniques such as polar modulation separate a signal into phase and amplitude components that are primarily frequency domain and time domain, respectively. Extracting those signal components and reconstructing them in a high-efficiency power amplifier combines many of the issues discussed in this article.