

The Yin-Yang of Matching: Part 1—Basic Matching Concepts

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This two-part article covers basic matching concepts and matching network topologies, emphasizing methods for obtaining the desired performance with networks that are realizable in practice

When an electrical signal propagates through media, a portion of the signal is reflected at the interface between sections with differing impedances. This is analogous to light reflection in optical systems. The

reflected signal may pose problems, and the power in the reflected signal reduces the transmitted power.

Figure 1 shows the impedance chart developed by Phillip Smith [1]. Any impedance with a positive real part may be displayed on the standard, unity radius Smith chart. The horizontal line is pure resistance. Circles with a center on this line are constant resistance. Arcs converging at center right are constant reactance. The top half of the chart is inductive and the bottom half is capacitive. The chart is sometimes normalized to the desired reference impedance of the system such as 50 ohms. For example, the inductive impedance $50 + j50$ ohms becomes $1 + j1$ when normalized and is plotted at the intersection of the circle labeled 1 and the top arc labeled 1. An admittance version of the chart is a left-right mirror of the impedance chart. A detailed description of the chart is given in Smith's book and a CD-ROM tutorial by Glenn Parker [2].

The center of the chart is the reference impedance of the system. An impedance at this point represents an ideal match. The length of a vector from the center of the chart to any impedance point is the magnitude of the reflection coefficient, $|\rho|$. The angle of the vector is from the center of the chart with

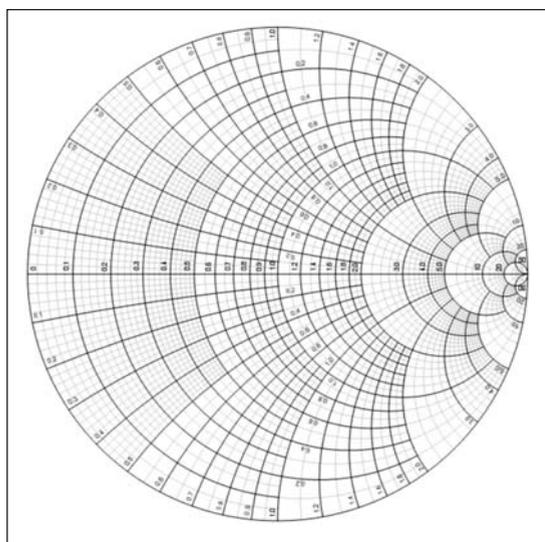


Figure 1 · The impedance Smith chart with unity radius.

respect to the real axis on the right. An ideal match has $|\rho| = 0$ and total reflection from a pure reactive impedance at the circumference of the chart has $|\rho| = 1$. The reflection coefficient may be computed from

$$\rho = \frac{Z - Z_0}{Z + Z_0} \quad (1)$$

where Z is the impedance and Z_0 is the reference impedance of the system.

The magnitude of the reflection coefficient is a radial parameter. In other words, it is a function of radial distance from the center of the chart. Other useful radial formats are related to ρ .

$$RL_{dB} = -20 \log |\rho| \quad (2)$$

	RL _{dB}	VSWR	L _A
0.025	32.0	1.051	0.003
0.050	26.0	1.105	0.011
0.075	22.5	1.162	0.024
0.100	20.0	1.222	0.044
0.125	18.1	1.286	0.068
0.150	16.5	1.353	0.099
0.175	15.1	1.424	0.135
0.200	14.0	1.500	0.177
0.225	13.0	1.581	0.226
0.250	12.0	1.667	0.280
0.300	10.5	1.857	0.410
0.350	9.1	2.077	0.568
0.400	8.0	2.333	0.757
0.450	6.9	2.636	0.983
0.500	6.0	3.000	1.249
0.600	4.4	4.000	1.938
0.700	3.1	5.667	2.924
0.800	1.9	9.000	4.437
0.900	0.9	19.000	7.212

Table 1 · Radial parameters. L_A accounts only for reflection loss and not dissipation loss.

$$VSWR = \frac{1 + |\rho|}{1 - |\rho|} \quad (3)$$

$$L_A = -10 \log(1 - |\rho|^2) \quad (4)$$

For example, in a 50 ohm system, an impedance 50 + j50 has a reflection coefficient of 0.4472 at 63.43°, a return loss of 7.0 dB, a VSWR of 2.618 and an insertion loss of 0.9691 dB. The insertion loss, L_A, is due to reflection. It does not account for dissipation loss that is present with lossy components or resistance in the circuit. Table 1 lists radial parameters. This table is useful for disciplines other than matching. For example, a filter designed for 0.177 dB passband ripple has a passband return loss of 15 dB.

Complex-Conjugate Match

Consider a load impedance of Z_L = R_L - jX_L at 100 MHz where R_L = 50 ohms and X_L = -33.86 ohms as shown in Figure 2. This load is matched to the 50 ohm source using a series

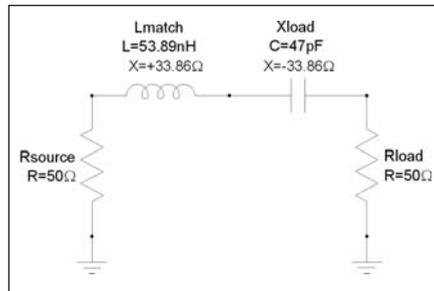


Figure 2 · A load of 50 - j33.86 ohms matched to a 50 ohm source using an inductor of reactance +33.86 ohms.

53.89 nH inductor with a reactance +33.86 ohms at 100 MHz. At this frequency the inductor and load reactance series resonate effectively connecting the 50 ohm source directly to the 50 ohm load resistance. Given in Figure 3 are the transmission and reflection responses of the original load and the matched load. The match is achieved only at the series resonant frequency of the load capacitance and matching inductance. In fact, the match is worse above 140 MHz than without the matching network. This is a characteristic of matching networks; the presence of reactive elements often provides selectivity.

The impedance seen looking toward the source through the matching inductor is 50 + j33.86 ohms. This is the complex-conjugate of the load impedance. In fact, at any node in a lossless matching network, the impedance to the left is the complex conjugate of the impedance to the right. This is another characteristic of matching networks.

If the load reactance is in parallel with the resistance, then a shunt element can cancel the load susceptance and provide a parallel resonant match. A single inductor or capacitor can provide a match only if the load and source resistance are equal, or near enough so to provide an adequate match. In general, we are not so fortunate and two matching elements are required.

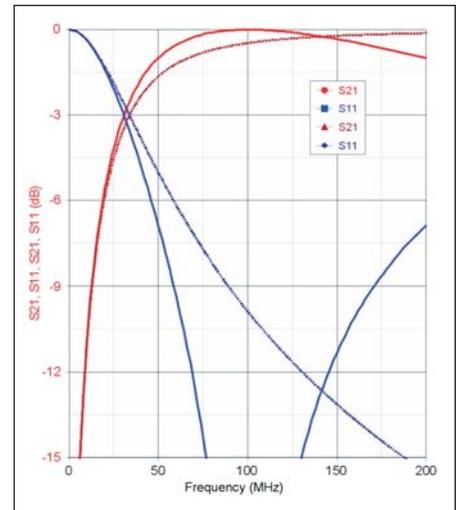


Figure 3 · Return loss and amplitude transmission responses of the complex-conjugate matched load (solid traces) and the original load without matching (dashed traces).

L-Network Matching

This section introduces L-networks for matching both real and complex impedances. At a single frequency, any positive-real complex impedance can be matched to any other positive-real complex impedance using no more than two reactive elements.

Consider the common need to match a complex impedance to a real impedance such as 50 ohms. Given in Figure 4 are the eight unique topologies comprised of two L-C elements. Each topology is capable of matching certain complex load terminations on the network's right to a real source resistance on the network's left. The charts are normalized to the desired source resistance. The matchable space is enclosed by the green curves in Figure 4. Notice these curves are the familiar shape of the Chinese Yin-Yang for the four topologies that include both an inductor and a capacitor. I first became aware of these concepts when reading Smith's book *Electronic Applications of the Smith Chart* [1]. Smith's unique ability to graphically express important concepts encompasses yin-yang!

Certain impedances are matched by only two topologies. For example, impedances with real components greater than the reference impedance are matched by only type 1 and 2 networks. Other impedances are matched by four topologies. For example, $Z = 0.5 + j1$ is matched by types 1, 4, 5 or 7.

Phil's book describes how to find the element values for these L-net-

works by using graphical techniques. Here are formulas for finding the element values analytically. For a reference impedance $Z_0 = R_0 - j0$, and a load termination impedance $Z_L = R_L - jX_L$, the load admittance is (eq. 5)

$$Y_L = G_L + jB_L = \frac{R_L}{R_L^2 + X_L^2} - j \frac{X_L}{R_L^2 + X_L^2}$$

then setting the following:

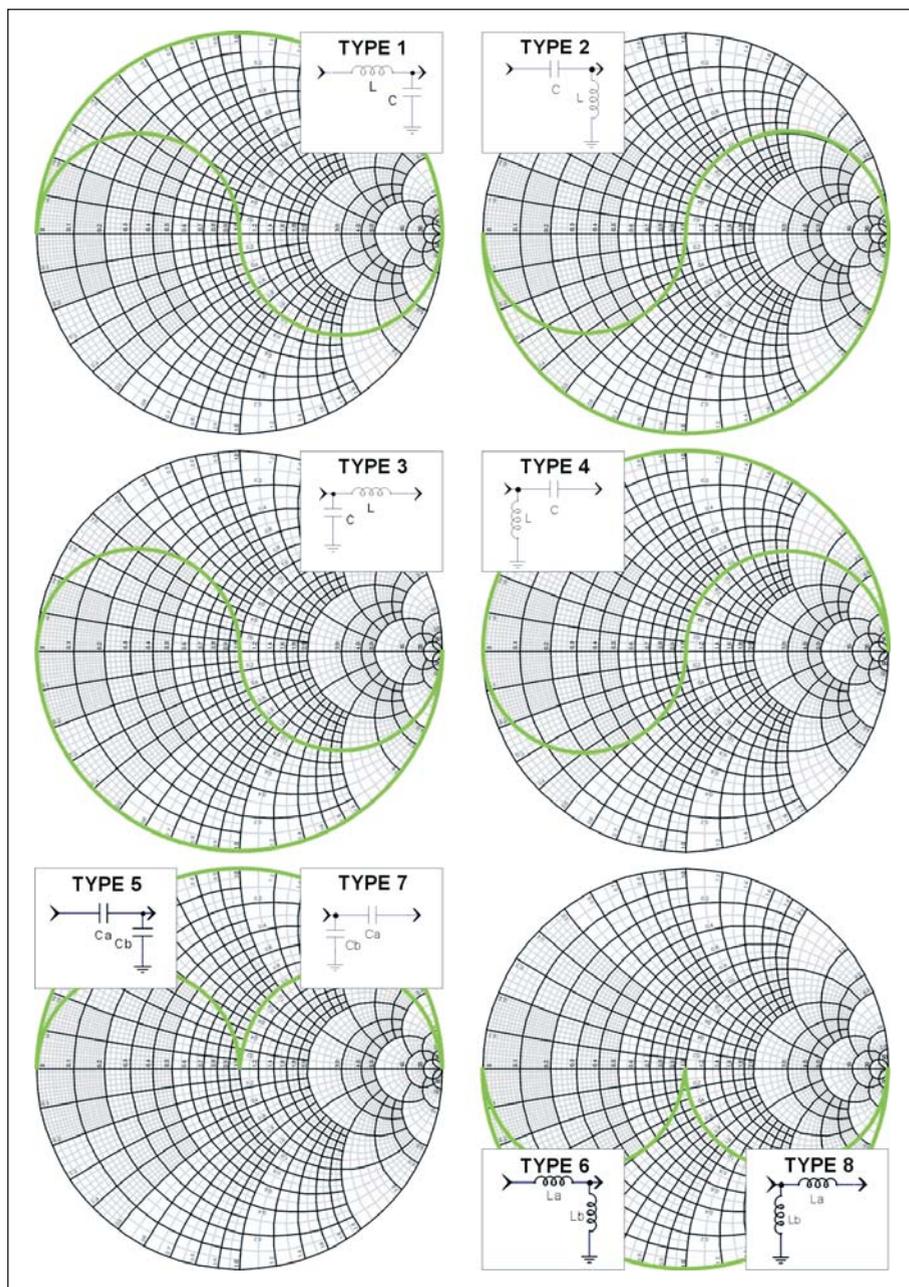


Figure 4 · Diagrams of matchable impedance space for L-networks of type 1 through 8.

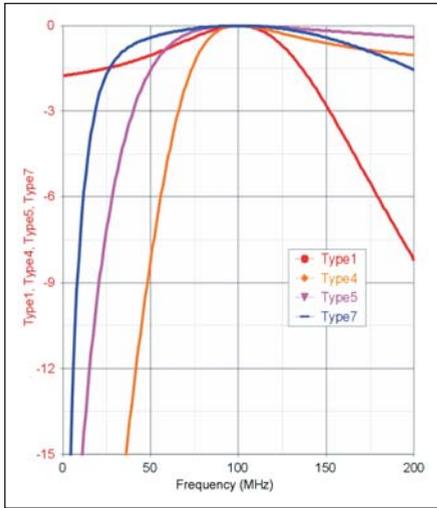


Figure 5 . Frequency domain responses of network types 1, 4, 5 and 7 that match $25 + j43.33$ ohms to 50 ohms.

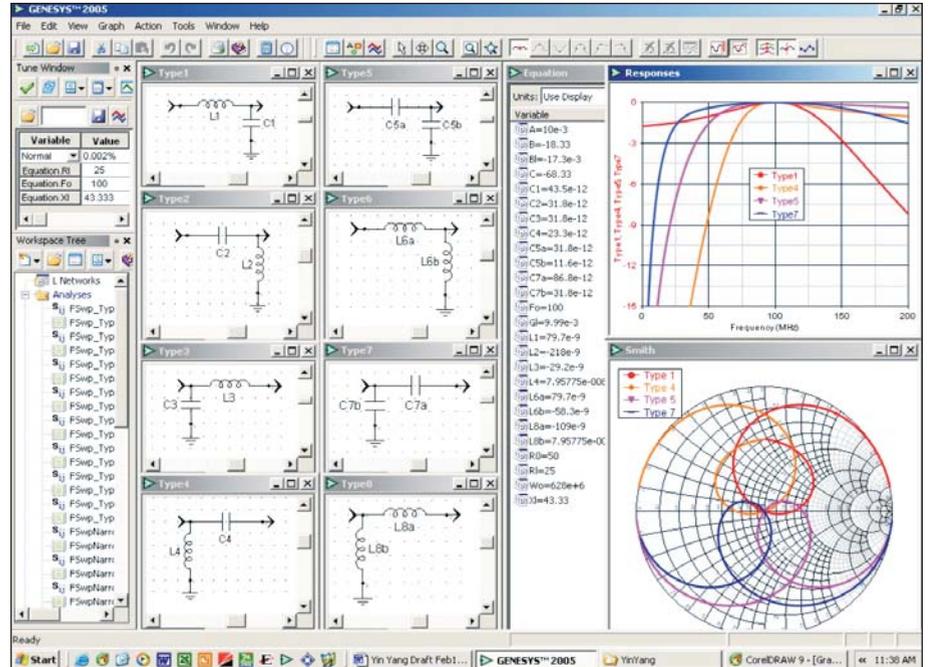


Figure 6 . Agilent GENESYS software screen depicting the design and responses of all eight L-network topologies.

$$A = \sqrt{\frac{G_L}{R_0} - G_L^2} \quad (6)$$

$$B = \sqrt{R_L(R_0 - R_L)} - X_L \quad (7)$$

$$C = -\sqrt{R_L(R_0 - R_L)} - X_L \quad (8)$$

we then have

$$L_1 = \frac{A}{2\pi f(G_L^2 + A^2)} \quad (9)$$

$$C_1 = \frac{A - B_L}{2\pi f} \quad (10)$$

$$L_2 = \frac{1}{2\pi f(A + B_L)} \quad (11)$$

$$C_2 = \frac{G_L^2 + A^2}{2\pi f A} \quad (12)$$

$$L_3 = \frac{B}{2\pi f} \quad (13)$$

$$C_3 = \frac{B - X_L}{2\pi f(R_L^2 + (B + X_L)^2)} \quad (14)$$

$$L_4 = -\frac{(R_L^2 + (C + X_L)^2)}{2\pi f(C + X_L)} \quad (15)$$

$$C_4 = -\frac{1}{2\pi f C} \quad (16)$$

$$C_{5a} = \frac{G_L^2 + A^2}{2\pi f A} \quad (17)$$

$$C_{5b} = -\frac{A - B_L}{2\pi f} \quad (18)$$

$$L_{6a} = \frac{A}{2\pi f(G_L^2 + A^2)} \quad (19)$$

$$L_{6b} = -\frac{1}{2\pi f(A - B_L)} \quad (20)$$

$$C_{7a} = -\frac{1}{2\pi f B} \quad (21)$$

$$C_{7b} = \frac{B - X_L}{2\pi f(R_L^2 + (B + X_L)^2)} \quad (22)$$

$$L_{8a} = \frac{C}{2\pi f} \quad (23)$$

$$L_{8b} = -\frac{(R_L^2 + (X_L + C)^2)}{2\pi f(X_L + C)} \quad (24)$$

Example

Consider a load termination impedance of $25 + j43.33$ ohms at 100 MHz. The return loss is 4.8 dB and the transmission loss is 1.76 dB. Referring to Figure 2 we see that type 1, 4, 5 or 7 is used to match this load. Eqs. 5 through 10 are used to find the element values for type 1: L_1 is 79.66 nH and C_1 is 43.47 pF.

Which of these topologies is best? The amplitude transmission responses are given in Figure 5 as computed and displayed by the Agilent GENESYS program suite [3]. Notice that all four provide full transmission at the design frequency, 100 MHz. Notice that the type 1 network is somewhat lowpass in character and that the others are somewhat highpass. The response characteristics are a function of the load impedance. In this case, type 7 provides the widest bandwidth. In general,

responses become more bandpass in character and narrower for larger load reflection coefficients. In some applications wide bandwidth is desired while for others the rejection of interfering signals by a narrow response is desired. The use of a simulator is a convenient way to simultaneously determine element values and compute network responses. Given in Figure 6 is a screen capture of an GENESYS workspace that computes element values and displays the responses for all eight L-networks. The user enters the reference impedance, load resistance, load reactance and center frequency as seen at the upper left. The computed element values are shown on the right. Notice that, for this load impedance, four of the L-networks have negative element values. These are depicted in the schematics as magenta colored inductors. If negative-valued inductors could be realized then all eight L-networks would function as matching networks. This GENESYS workspace may be downloaded from the online post “Matching Tutorial Published in *High Frequency Electronics Magazine*” at the Founder’s Forum at www.eagleware.com.

Distributed Networks

Component values in all matching networks become extreme as the reflection coefficient of the load approaches 1.0. Realizability is then at risk. These issues are even more critical with distributed circuits because of the difficulty in realizing transmission lines with high and low characteristic impedance. The property of reentrance, where transmission line behavior repeats at multiples of 90° or 180°, restricts the use of distributed circuits when good stopbands are desired. Nevertheless, distributed networks are useful for matching.

A microstrip transmission line involves inductance associated with the flow of current in the conductor and capacitance associated with the strip separated from ground by the

dielectric substrate. This distributed inductance and capacitance is the basis of the classic L-C model for a transmission line and it accounts for the term “distributed.” If the line is narrow, the inductance is increased but the capacitance is small. A narrow, high-impedance line behaves like an inductor if it is less than 90 degrees in electrical length. A wide,

low-impedance line looks capacitive.

Lets examine this analytically. The impedance, Z_S , at the input of a transmission line of characteristic impedance Z_0 and length θ terminated in a load, Z_L , is given by

$$Z_S = Z_0 \frac{Z_L - jZ_0 \tan \theta}{Z_0 + jZ_L \tan \theta} \quad (25)$$

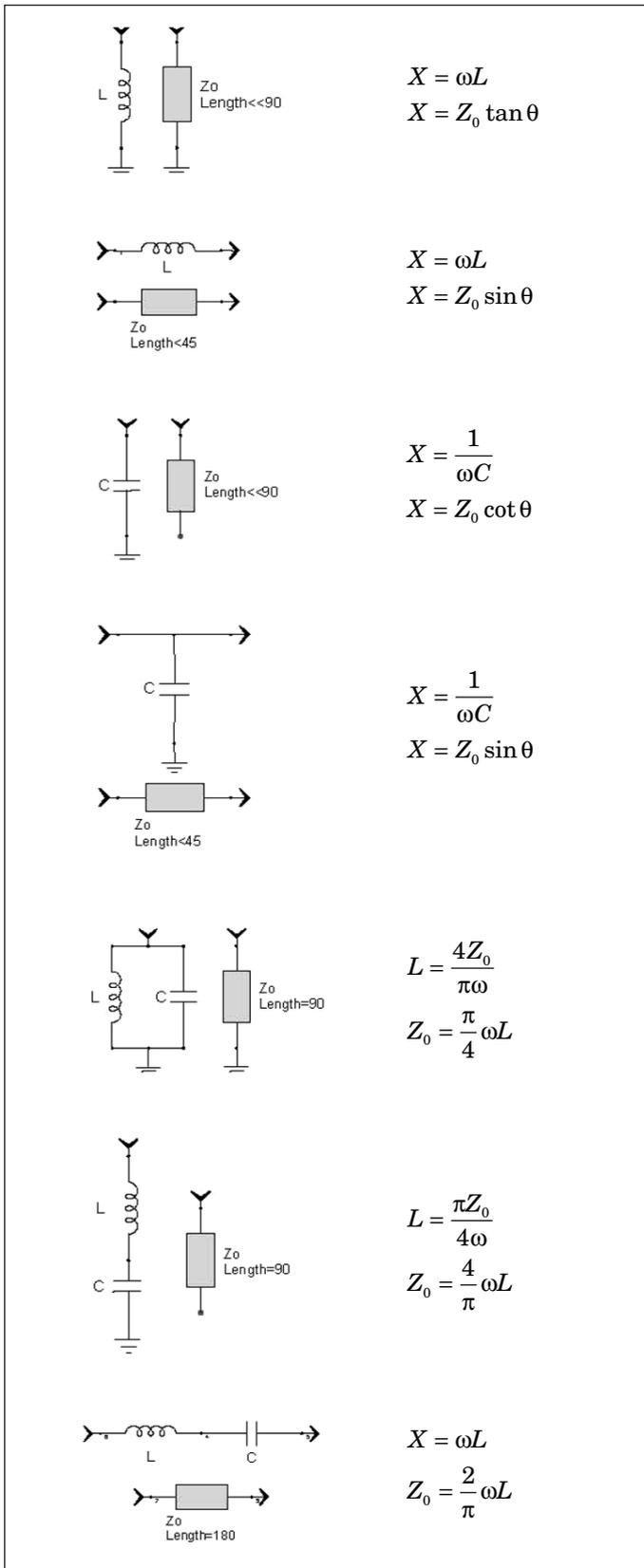


Figure 7 · Equivalent relationships between lumped and distributed elements and resonators.

Consider the case where Z_L is a short, $0 + j0$. Then

$$Z_s = jZ_0 \tan \theta \tag{26}$$

Since the input impedance of a shorted inductor is jX_L ,

$$X_L = Z_0 \tan \theta \tag{27}$$

This means that the reactance of an inductor in a network may be replaced with a transmission line of characteristic impedance Z_0 and length θ . Figure 7 presents the equivalent relationships between lumped and distributed circuits. These equivalents are exact only at the design frequency. The reactance of an inductor increases linearly with increasing frequency while the reactance of a shorted line increases as $\tan \theta$. If the line is short, that is $\theta \ll 90^\circ$, then $\tan \theta \approx \theta$ and the input reactance of a shorted line increases linearly with frequency. Therefore, a shorted line behaves like an inductor over a range of frequencies where the line is much less than 90° long, preferably less than 30° .

The relationships in Figure 7 may be used to replace lumped elements with distributed elements. These equivalents are useful in circuits other than matching networks such as filters, oscillators, and couplers. Generally, the equivalence is better with higher impedance and shorter length lines for inductors, and with lower impedance and shorter length lines for capacitors. For example, a 47 nH inductor has a reactance of 29.53 ohms at 100 MHz. A shorted 30.57° , 50 ohm line provides the same reactance. However, a shorter (16.45°) 100 ohm line would behave like this inductor over a wider frequency range. For the 180° resonator, the line impedance should be either higher or lower impedance than the impedance of the system it is inserted within. The 90° resonators have unique impedance values.

Distributed L-Networks

Conspicuously absent in Figure 7 is a transmission line equivalent for a series capacitor. Series capacitors are difficult to realize in distributed form. A microstrip gap requires intolerably close spacing to achieve significant capacitance and the line ends have parasitic capacitance to ground as well as the series capacitance. Edge coupled lines have even larger capacitance to ground. Because of this, the most popular form of distributed highpass filters use transmission lines for the shunt inductors and lumped-element series capacitors.

However, from Figure 4, we can see that L-network types 1, 3, 6 and 8 do not require series capacitors. Using the equivalents in Figure 7, these types may be realized in distributed form.

In summary, once the lumped values are computed,

$$Z_0(\text{SeriesLines}) = \frac{\omega L_{\text{Lumped}}}{\sin \theta} \quad (28)$$

$$Z_0(\text{Shunt Shorted Stub}) = \frac{\omega L_{\text{Lumped}}}{\tan \theta} \quad (29)$$

$$Z_0(\text{Shunt Open Stub}) = \frac{\tan \theta}{\omega C_{\text{Lumped}}} \quad (30)$$

$$\theta_{10a} = \tan^{-1} \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (37)$$

Next the required susceptance of the stub is found by

$$B_s = \frac{(R_L'^2 + X_L' \tan \theta_{\text{series}} + X_L'^2 - 1) \tan \theta_{\text{series}} - X_L'}{R_L'^2 + (X_L' + \tan \theta_{\text{series}})^2} \quad (38)$$

Then

$$\theta_{9b} = \tan^{-1} \frac{1}{B_s} \quad (39)$$

$$\theta_{10b} = -\tan^{-1} B_s \quad (40)$$

Suppose we desire to match an antenna with an input impedance of $35.5 - j107$ ohms to 50 ohms using the single-stub tuner. Then the series 50 ohm line is 47.8° long and the shorted 50 ohm stub is 21.3° long.

Next Issue

In the next issue, this article concludes with Part 2, covering the topics of impedance transformation through transmission line sections, multiple-section transformers, absorbing reactance using filters, and analysis of load characteristics for selection of the preferred matching method.

Author Information

Randall Rhea received a BSEE from the University of Illinois and and MSEE from Arizona State. He worked at the Boeing Company, Goodyear Aerospace and Scientific-Atlanta. He is the founder of Eagleware Corporation which was acquired by Agilent Technology in 2005 and Noble Publishing which was acquired by SciTech Publishing in 2006. He has authored numerous papers and tutorial CDs, the books *Oscillator Design and Computer Simulation* and *HF Filter Design and Computer Simulation* and has taught seminars on oscillator and filter design.

References

1. P. Smith, *Electronic Applications of the Smith Chart*, 2nd edition, 1995, SciTech/Noble Publishing, Raleigh, North Carolina.
2. G. Parker, *Introduction to the Smith Chart*, (CD-ROM tutorial), 2003, SciTech/Noble Publishing, Raleigh, North Carolina.
3. Agilent Technologies, EEsof EDA Division, Santa Rosa, CA. www.agilent.com/find/eesof
4. G. Matthaei, L. Young, and E.M.T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling*

Single-Stub Tuner

Given in Figure 8 are single-stub tuners with shorted and open stubs and their matchable space plotted on the Smith chart. The shorted-stub tuner is similar to L-network type 8, while the open-stub tuner is similar to type 3. However, by using transmission line theory embodied in Eq. 25, a more general solution is achieved that does not require short line lengths. Also, the more general solution matches a larger space than a lumped Type I network. The formulas given here are for the case where the characteristic impedance of the lines in the network equal the reference impedance of the system. This is convenient because, if a match to 50 or 75 ohms is desired, lines of that impedance are typically available. Allowing other transmission line characteristic impedances may provide wider bandwidth.

The following equations are more concise when the load impedance is normalized to Z_0 .

$$Z_L' = \frac{Z_L}{Z_0} = R_L' + jX_L' \quad (31)$$

Then, the length of the series transmission line for type 9 networks, θ_{9a} , is

$$\theta_{9a} = \tan^{-1} \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (32)$$

$$\text{if } \theta_{9a} < 0^\circ \text{ then } \theta_{9a} = \theta_{9a} + 180^\circ \quad (33)$$

where

$$a = 1 - R_L' \quad (34)$$

$$B = 2X_L' \quad (35)$$

$$c = R_L'^2 + X_L'^2 - R_L' \quad (36)$$

The length of the series line for type 10 networks is

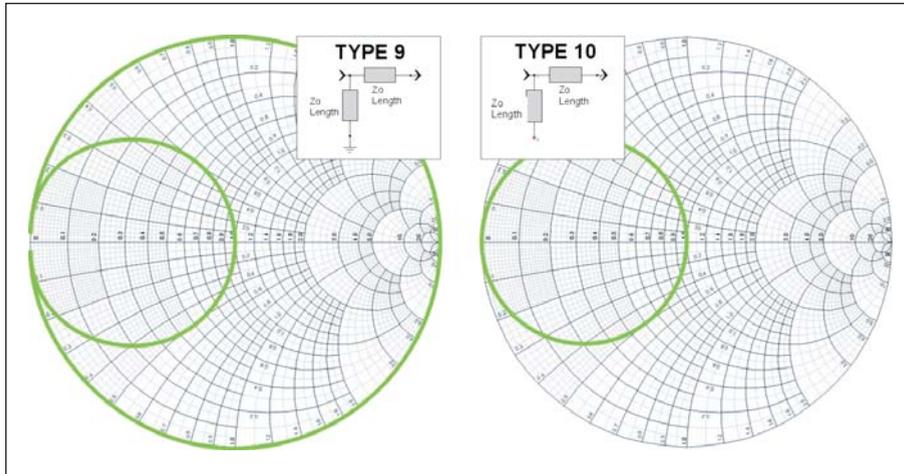


Figure 8 - On the left is a single-stub transmission-line tuner with a shorted stub and on the right is a tuner with an open stub. The matchable space of the shorted stub includes much of the chart but excludes the smaller unity conductance circle on the left.

Structures, Artech House, Dedham, MA, 1967/1980.

5. R. Rhea, *Practical Issues in RF Design* (3 set CD-ROM tutorial),

2003, SciTech/Noble Publishing, Raleigh, North Carolina.

6. R. M. Fano, "Theoretical Limitations on the Broadband Matching of

Arbitrary Impedances," *Journal of the Franklin Institute*, January 1950.

7. R. Rhea, *Filter Techniques* (3 set CD-ROM tutorial), 2003, SciTech/Noble Publishing, Raleigh, North Carolina.

8. R. Levy, "Explicit Formulas for Chebyshev Impedance-Matching Networks," *Proc. IEEE*, June 1964.

9. T.R. Cuthbert, Jr., *Circuit Design Using Personal Computers*, John Wiley, New York, 1983.

10. J. Sevick, *Transmission Line Transformers*, 4th ed., 2001, SciTech/Noble Publishing, Raleigh, North Carolina.

11. J. Sevick, "Design of Broadband Ununs with Impedance Ratios Less Than 1:4," *High Frequency Electronics*, November, 2004.

12. J. Sevick, "A Simplified Analysis of the Broadband Transmission Line Transformer," *High Frequency Electronics*, February, 2004.