Exact Time-Domain Analysis of Class E Power Amplifiers with Quarterwave Transmission Line

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Abstract – The results of exact time domain analysis of the switched-mode tuned Class E power amplifiers with a quarterwave transmission line are presented with analytical determination of the optimum load network parameters. Effects of the quarterwave transmission line and collector capacitance on the current and voltage waveforms are discussed and analyzed. The ideal collector voltage and current waveforms demonstrate a possibility of 100-percent efficiency without overlapping between each other. A possibility of load network implementation including output matching circuit at RF and microwave frequencies using lumped and transmission line elements is considered with accurate derivation of the circuit parameters. The switched-mode Class E power amplifiers with a quarterwave transmission line offer a new challenge for RF and microwave power amplification providing high-efficiency operation conditions.

I. Introduction

The switched-mode tuned Class E power amplifiers with a shunt capacitance have found widespread application due to their design simplicity and high efficiency. These power amplifiers are widely used in different frequency ranges and at output power levels ranging from several kilowatts at low RF frequencies up to about one watt at microwaves. In the Class E power amplifier, the transistor operates as an on-to-off switch and the shapes of the current and voltage waveforms provide a condition where the high current and high voltage do not overlap simultaneously. This minimizes the power dissipation and maximizes the power amplifier efficiency. The possibility of increasing the efficiency of the single-ended power amplifier by mistuning the output matching circuit was found quite a long ago [1]. However, the single-ended switched-mode power amplifier with a shunt capacitance as a Class E power amplifier realizing ideally 100-percent efficiency was first introduced in 1975 [2].

The characteristics of a Class E power amplifier can be determined by finding its steady-state collector voltage and current waveforms. The basic circuit of a class E power amplifier with a shunt capacitance is shown in Figure 1 where the load network consists of a capacitance C shunting the transistor, a series inductance L, a series fundamentally tuned L_0C_0-circuit and a load resistance R. In a common case, the shunt capacitance C can represent the intrinsic device output capacitance and external circuit capacitance added by the load network. The collector of the transistor is connected to the supply voltage by RF choke having high reactance at the fundamental frequency. The active device is considered to be an ideal switch that is driven in such a way to provide the device switching between its on-state and off-state operation conditions. As a result, the collector voltage waveform is determined by the switch when it is on and by the transient response of the load network when the switch is off.

To simplify an analysis of a switched-mode Class E power amplifier, the following several assumptions are introduced:

- the transistor has zero saturation voltage, zero saturation resistance, infinite off-resistance and its switching action is instantaneous and lossless;
- the total shunt capacitance is independent of the collector and is assumed to be linear;
- the RF choke allows only a constant DC current and has no resistance;
- the loaded quality factor Q_L of the series resonant L_0C_0-circuit tuned on the fundamental frequency $\omega = 1/\sqrt{L_0C_0}$ is high enough in order the output current to be sinusoidal at the switching frequency;
- there are no losses in the circuit except only into the load R;
- for optimum operation mode a 50% duty cycle is used.

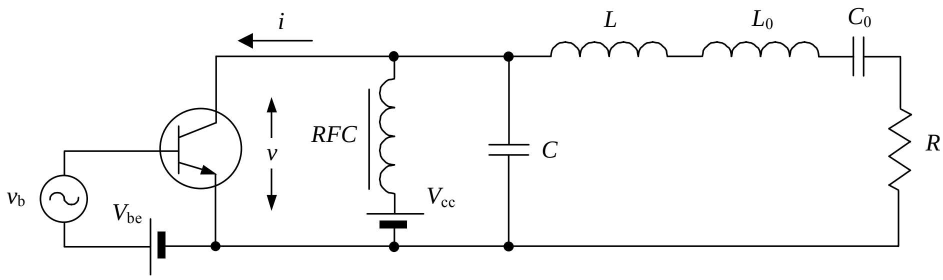


Fig. 1. Equivalent circuit of Class E power amplifier with shunt capacitance

For lossless operation mode, it is necessary to provide the following optimum conditions for voltage across the switch just prior to the start of switch on at the moment $t = 2\pi$, when transistor is saturated:

$$v(\omega t)\big|_{\omega t=2\pi} = 0, \qquad (1)$$

$$\left.\frac{dv(\omega t)}{d\omega t}\right|_{\omega t=2\pi} = 0, \qquad (2)$$

where v is the voltage across the switch.

In Figure 2, the normalized (*a*) collector voltage waveform and (*b*) collector current waveforms for idealized optimum Class E with a shunt capacitance are shown. From collector voltage and current waveforms it follows that, when the transistor is turned on, there is no voltage across the switch and the current i consisting of the load sinusoidal current and DC current flows through the device. However, when the transistor is turned off, this current now flows through the parallel capacitance C.

As a result, the optimum series inductance L and shunt capacitance C can be found from [3]

$$L = 1.1525\frac{R}{\omega}, \tag{3}$$

$$C = \frac{0.1836}{\omega R}, \tag{4}$$

whereas the optimum load resistance R can be obtained for the given supply voltage V_{cc} and output power P_{out} by

$$R = 0.5768\frac{V_{cc}^2}{P_{out}}. \tag{5}$$

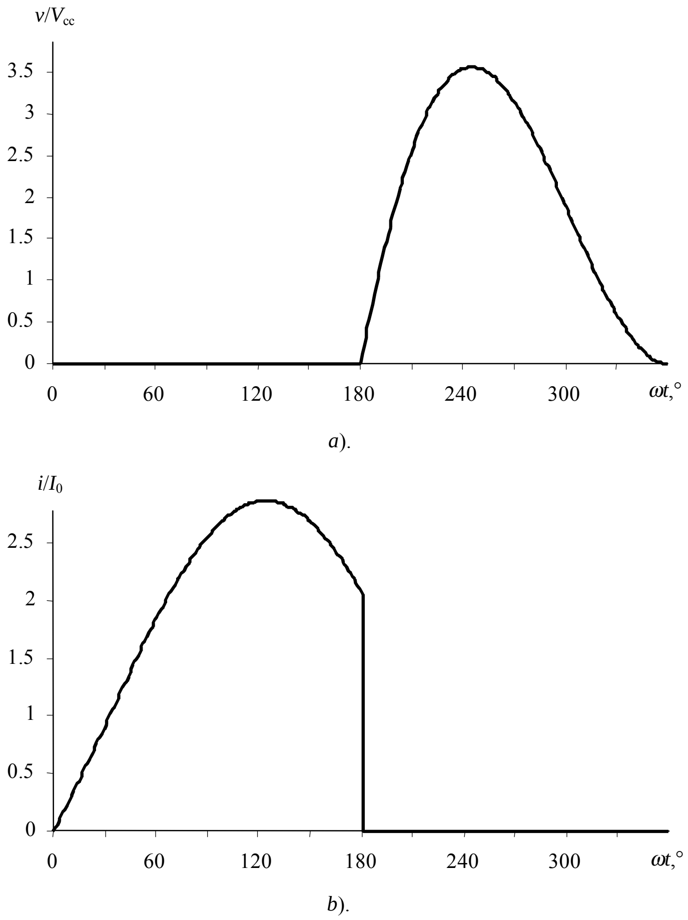


Fig. 2. Collector voltage and current waveforms for idealized optimum Class E with shunt capacitance

For a microwave power amplifier, usually all inductances in its output matching circuit should be realized by means of the transmission lines to reduce the power losses. As a result, to approximate the idealized Class E mode with a shunt capacitance, it is necessary to design the transmission-line load network with required idealized optimum impedance at the fundamental given by

$$Z_{net1} = R\left(1 + j\tan 49.052^\circ\right), \tag{6}$$

where Z_{net1} is the fundamental load network impedance seen from the device collector [4]. At the same time, the open circuit conditions should be provided for all harmonic components. However, as it turned out from the Fourier analysis, a good approximation to Class E mode with a shunt capacitance may be obtained with the collector voltage waveform across the switch consisting only of the fundamental and second harmonic components [4]. In Figure 3(*a*), the voltage waveform containing the fundamental and second harmonic only (dotted line) is plotted along with the ideal one containing all harmonics (solid line). Using such an approach, the Class E power amplifier with series microstrip line l_1 and open circuit stub l_2, whose equivalent circuit is shown in Figure 3(*b*), was designed for microwave applications. The electrical lengths of lines l_1 and l_2 are chosen to be of about 45° at the fundamental to provide an open circuit condition at the second harmonic, whereas their characteristic impedances are calculated to satisfy the required inductive impedance condition at the fundamental. The output lead inductance of the packaged device can be accounted for by a shortening the length of l_1.

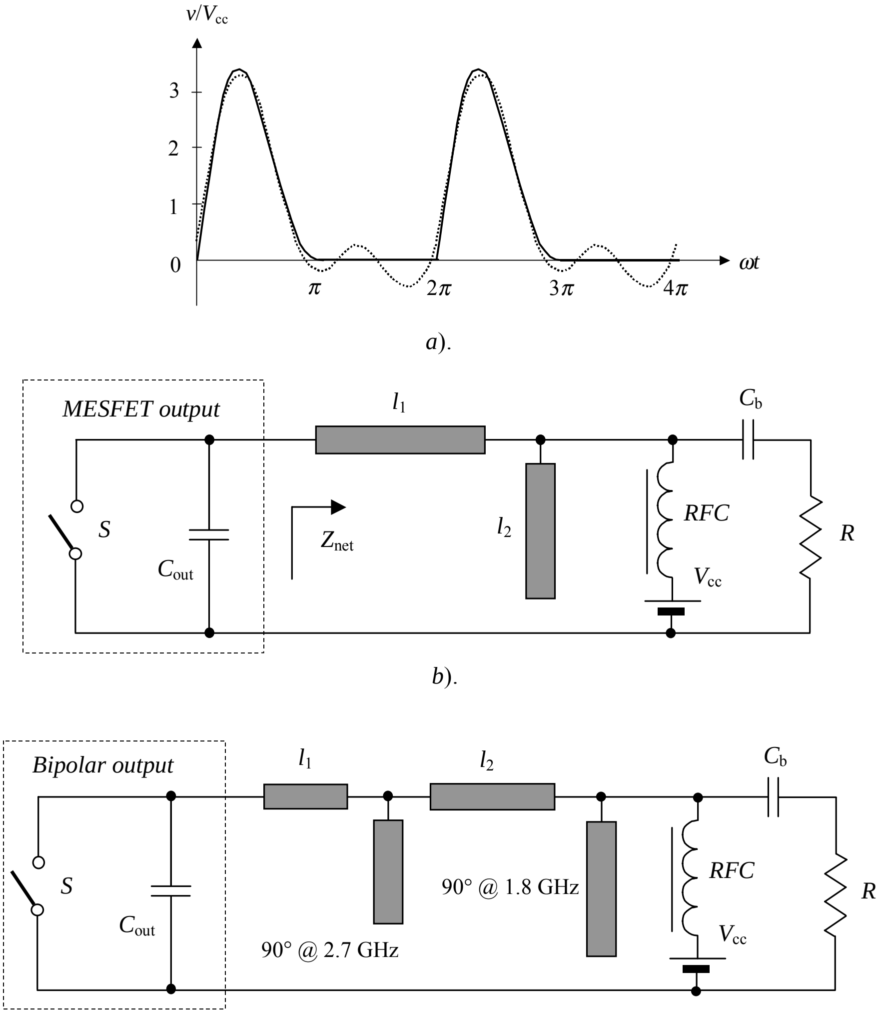


Fig. 3. Two-harmonic voltage waveform and equivalent circuits of Class E power amplifiers with transmission lines

An additional increase of the collector efficiency can be provided by the load impedance control at the second and third harmonics simultaneously [5]. Such a harmonic control network consists of the open circuit quarterwave stubs both at the second harmonic and third one separately. As shown in Figure 3(*c*), the third-harmonic quarterwave stub is located before the second-harmonic one. It is possible to achieve very high collector efficiency even with values of the device output capacitance higher than it is conventionally required at the expense of lower output power keeping the load at the second and third harmonics strictly inductively reactive. As a result, maximum collector efficiency over 90% for power amplifier with the output power of 1.5 W can be realized at 900 MHz.

II. QUARTERWAVE TRANSMISSION LINE

It is known that using a quarterwave transmission line in the load network can significantly improve the power amplifier efficiency. For example, to realize a 100-percent idealized collector efficiency in Class F operation, it is required to provide the following impedance conditions for the fundamental and harmonic components at the device collector [6, 7]:

$$\begin{cases} Z_1 = R = \dfrac{8}{\pi^2}\dfrac{V_{cc}}{I_0} \\ Z_n = 0 \quad \text{for even } n \\ Z_n = \infty \quad \text{for odd } n \end{cases} \tag{7}$$

where V_{cc} is the collector supply voltage, I_0 is the collector DC current, R is the load resistance at the fundamental, n is the harmonic number.

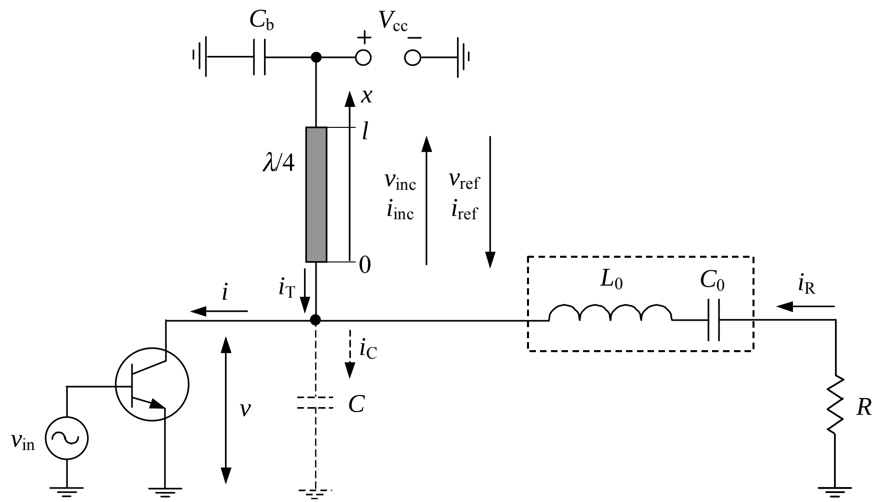


Fig. 4. Basic circuit of a class F power amplifier with quarterwave transmission line

The basic circuit of a class F power amplifier is shown in Figure 4 where the load network consists of a parallel quarterwave transmission line RF grounded at the end by bypass capacitor, a series fundamentally tuned L_0C_0-circuit and a load resistance R. In an idealized case, the intrinsic device output capacitance is assumed to be negligible to affect the power amplifier RF performance. The loaded quality factor Q_L of the series resonant L_0C_0-circuit should be high enough to provide sinusoidal output current flowing into the load R.

To define the collector voltage and current waveforms, consider the electrical behaviour of a homogenious lossless quarterwave transmission line connected to the DC voltage supply with RF grounding. In this case, the voltage $v(t, x)$ in any cross section of such a transmission line can be represented as a sum of the incident and reflected voltages, $v_{inc}(\omega t-2\pi x/\lambda)$ and $v_{refl}(\omega t+2\pi x/\lambda)$, generally with an arbitrary wavefom. When $x = 0$, the voltage $v(t, x)$ is equal to the collector voltage:

$$v(\omega t) = v(t, 0) = v_{inc}(\omega t) + v_{refl}(\omega t). \tag{8}$$

At the same time, at another end of the transmission line when $x = \lambda/4$, the voltage is constant and equal to

$$V_{cc} = v(t, \pi/2) = v_{inc}(\omega t-\pi/2) + v_{refl}(\omega t+\pi/2). \tag{9}$$

Since the time moment t was chosen arbitrary, let us rewrite equation (9) using a phase shift of $\pi/2$ for each voltage by

$$v_{inc}(\omega t) = V_{cc} - v_{refl}(\omega t+\pi). \tag{10}$$

Substituting equation (10) into equation (8) yields

$$v(\omega t) = v_{refl}(\omega t) - v_{refl}(\omega t+\pi) + V_{cc.} \tag{11}$$

Consequently, for the phase shift of π, the collector voltage can be obtained by

$$v(\omega t+\pi) = v_{refl}(\omega t+\pi) - v_{refl}(\omega t+2\pi) + V_{cc.} \tag{12}$$

For idealized operation condition with a 50-percent duty cycle when during half a period transistor is on and during another half a period the transistor is off with overall period of 2π, the voltage $v_{refl}(\omega t)$ can be considered as the periodical function with a period of 2π:

$$v_{refl}(\omega t) = v_{refl}(\omega t+2\pi). \tag{13}$$

As a result, the summation of equations (11) and (12) results in the required equation for collector voltage:

$$v(\omega t) = 2V_{cc} - v(\omega t+\pi). \tag{14}$$

From equation (14) it follows that for any t when $v \geq 0$, maximum value of the collector voltage cannot exceed a value of $2V_{cc}$ and the time duration with maximum voltage of $v = 2V_{cc}$ coincides with the time duration with minimum voltage of $v = 0$.

Similarly, equation for current i_T flowing into the quarterwave transmission line can be obtained by

$$i_T(\omega t) = i_T(\omega t+\pi), \tag{15}$$

which means that the period of signal flowing into the quarterwave transmission line is equal to π because it contains only even harmonic components as such a transmission line has an infinite impedance at odd harmonics.

Hence, to completely realize the impedance conditions for ideal Class F operation mode given by equation (7), a quarterwave transmission line is required to provide the open circuit conditions for odd harmonics and short circuit conditions for even harmonics. At the same time, the series fundamentally tuned L_0C_0-circuit with infinite loaded quality factor Q_L is necessary to provide the sinusoidal fundamental current flowing into the load (see Figure 5(*a*)) realizing also open circuit conditions at odd harmonics.

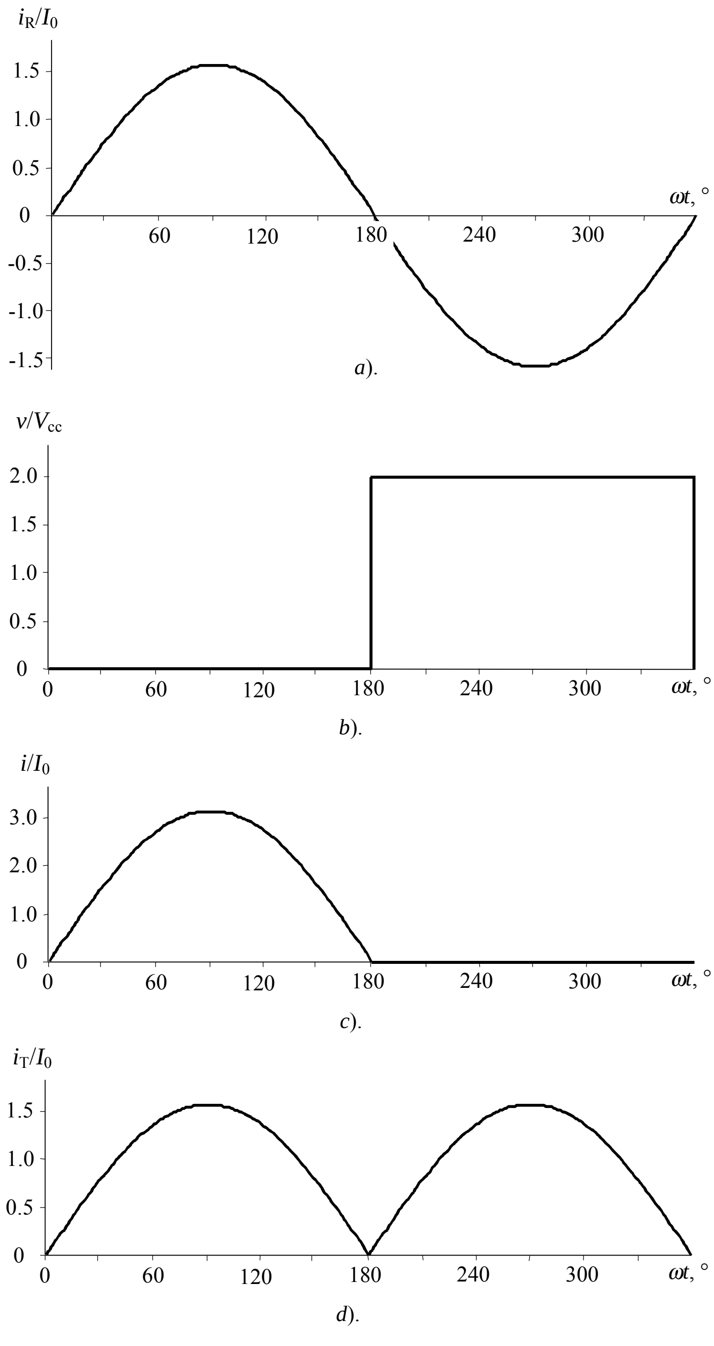


Fig. 5. Ideal waveforms of Class F power amplifier with quarterwave transmission line

Consider the transistor as an ideal switch when it is closed during the interval $0 < \omega t \le \pi$ where $v = 0$ and open during the interval $\pi < \omega t \le 2\pi$ where $v = 2V_{cc}$ according to equation (14). During the interval $\pi < \omega t \le 2\pi$ when switch is open, the load is directly connected to the transmission line and $i_T = -i_R = -I_R \sin\omega t$. Consequently, during the interval $0 < \omega t \le \pi$ when switch is closed, $i_T = I_R\sin\omega t$ according to equation (15). Hence, the current flowing into the quarterwave transmission line at any ωt can be represented by

$$i_T(\omega t) = I_R\left|\sin\omega t\right|, \tag{16}$$

where I_R is the amplitude of current flowing into the load.

Since the collector current is defined as $i = i_T + i_R$, then

$$i(\omega t) = I_R(\sin\omega t + \left|\sin\omega t\right|), \tag{17}$$

which means that the collector current represents half-sinusoidal pulses with the amplitude equal to double load current amplitude.

The ideal collector voltage and current waveforms corresponding to Class F operation mode with a quarterwave transmission line are shown in Figure 5(*b*) and 5(*c*), respectively. Here, a sum of odd harmonics gives a square voltage waveform and a sum of the fundamental and even harmonics approximates a half-sinusoidal collector current waveform. An appropriate waveform corresponding to the normalized current flowing into the quarterwave transmission line shown in Figure 5(*d*) represents a sum of even harmonics.

III. Influence of Shunt Capacitance

In practice, the idealized collector voltage and current waveforms can be realized at low frequencies when effect of the device collector capacitance is negligible. At higher frequencies, the effect of the collector capacitance contributes to a finite switching time resulting in the time periods when the collector voltage and collector current exist at the same time, i.e. simultaneously $v > 0$ and $i > 0$. As a result, such a load network with shunt capacitance cannot provide the switched-mode operation with an instantaneous transition from the device pinch-off to saturation mode. Therefore, during a finite time interval the device operates in active region as a current source with the reverse-biased collector-base junction and the collector current is provided by this current source.

Equation for the current flowing through the collector capacitance can be obtained from equation (14) taking into account that voltage v is the voltage across the capacitance C:

$$i_C(\omega t) = -\, i_C(\omega t+\pi), \tag{18}$$

i.e., the current due to the capacitance charging process is equal to the current due to the capacitance discharging process with opposite sign and duration of charging and discharging periods are equal.

From Figure 4 it follows that the current flowing through the collector capacitance in the arbitrary time moment t can be written as

$$i_C(\omega t) = i_T(\omega t) + i_R(\omega t) - i(\omega t), \tag{19}$$

whereas, in the time moment $(t + \pi/\omega)$, it can be defined by

$$i_C(\omega t + \pi) = i_T(\omega t + \pi) + i_R(\omega t + \pi) - i(\omega t + \pi). \tag{20}$$

Let the output current flowing into the load is written as sinusoidal by

$$i_R(\omega t) = I_R \sin(\omega t + \varphi), \tag{21}$$

where φ is the initial phase shift due to the finite value of the collector capacitance.

Then, taking into account equations (15) and (18), from equation (20) it follows:

$$-i_C(\omega t) = i_T(\omega t) - i_R(\omega t) - i(\omega t + \pi). \tag{22}$$

Adding equation (19) and equation (22) yields

$$i(\omega t) + i(\omega t + \pi) = 2i_T(\omega t), \tag{23}$$

which specifies the relatioship in time domain between the collector current and current flowing into the transmission line.

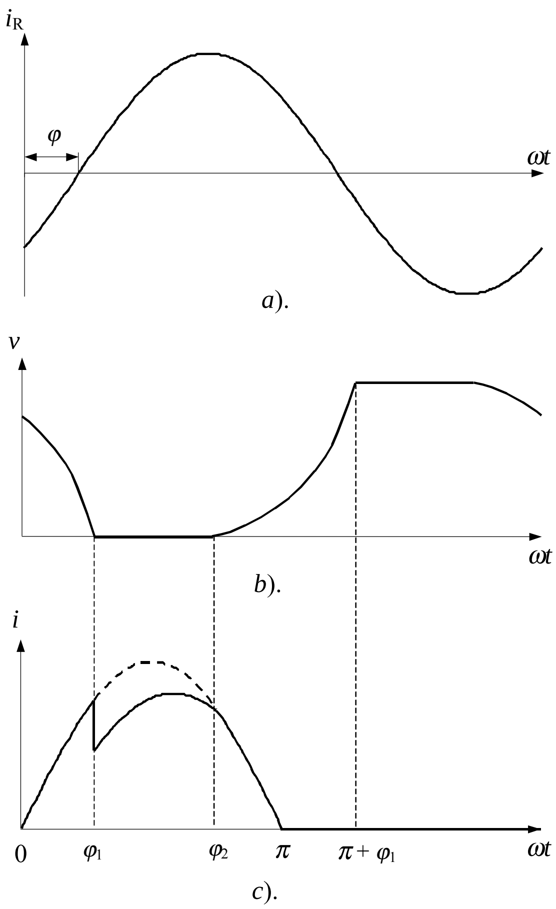


Fig. 6. Effect of shunt capacitance on voltage and current waveforms

The collector current and voltage waveforms are shown in Figure 6, where the phase angle φ_1 corresponds to the beginning of the transistor saturation mode, whereas the phase angle φ_2 corresponds to the beginning of the active mode and collector capacitance charging process start-up. During the saturation interval when $\varphi_1 < \omega t < \varphi_2$, the collector current i can be defined using equations (19), (21) and (22) by

$$i(\omega t) = i_T(\omega t) + i_R(\omega t) = 2i_R(\omega t) = 2I_R \sin(\omega t + \varphi). \tag{24}$$

In active region when $0 \le \omega t \le \varphi_1$ and $\varphi_2 \le \omega t \le \pi$, the collector current flowing into the load network is defined by the input driving signal and, for conduction angle of 180 degree, represents the periodic half-sinusoidal pulses written by

$$i(\omega t) = I_{\text{active}}\left(\sin\omega t + \left|\sin\omega t\right|\right), \tag{25}$$

where the collector current amplitude I_{active} in active region has a higher value than that one in saturation mode when $I_{\text{sat}} = 2I_{\text{R}}$ (see equation (24)) due to the shunting effect of the forward-biased collector-base diode junction when device is saturated. The moment of the opening of the collector-base junction corresponds to the time moment φ_1 with instantaneous reduction in the collector current waveform shown in Figure 6(*c*). Physically, this effect can be explained by the carrier injection from the device collector to its base region as a result of forward-biasing the collector-base junction. This saturation period is characterized by the diffusion capacitance of the forward-biased collector-base junction whereas, in active or pinch-off regions, the reverse-biased collector-base junction is described by the junction capacitance, which value is substantially smaller. The saturation period is ended in the time moment φ_2 corresponding to the beginning of the active mode and collector junction capacitance charging process start-up.

Using equations (16), (23) and (25), the current flowing into the transmission line i_{T} and current flowing through the collector capacitance i_{C} can be determined by

$$i_{\text{T}}(\omega t) = I_{\text{active}}\left|\sin\omega t\right|, \tag{26}$$

$$i_{\text{C}}(\omega t) = 2\left[I_{\text{R}}\sin(\omega t + \varphi) - I_{\text{active}}\sin\omega t\right]. \tag{27}$$

Power loss due to the charging and discharging processes of the device collector capacitance can be calculated from

$$P_{\text{loss}} = \frac{1}{2\pi}\int_0^{\varphi_1} v(\omega t)\, i(\omega t)\, d\omega t + \frac{1}{2\pi}\int_{\varphi_2}^{\pi} v(\omega t)\, i(\omega t)\, d\omega t \tag{28}$$

where the collector voltage v coincides with the voltage across the capacitance C. From equation (28) it follows that the longer active region due to the larger collector capacitance, the more losses and less efficient operation conditions of the power amplifier. The results of numerical calculations allowed to determine the maximum operating frequency where the collector efficiency of Class F power amplifier including the effect of the collector capacitance C is higher than the collector efficiency of conventional Class B power amplifier [8]:

$$f_{\max} \cong \frac{0.47}{RC} \tag{29}$$

where R is the load resistance.

IV. CLASS E MODE WITH QUARTERWAVE TRANSMISSION LINE

However, the collector efficiency can be increased and effect of the collector capacitance can be compensated with an inclusion of a series inductance between the shunt capacitance and quarterwave transmission line realizing Class E operation conditions. The possibility to include a quarterwave transmission line into the Class E load network (see Figure 1) instead of RF choke was firstly considered in [9]. However, such a location of a quarterwave transmission line with direct connection to the device collector violates the required capacitive impedance conditions at even harmonics by providing simply their shortening. As a result, an optimum Class E operation mode when the shapes of the collector current and voltage waveforms provide a condition when the high current and high voltage do not overlap simultaneously cannot be realized. In this case, the effect of parallel connection of the quarterwave transmission line and shunt capacitance results in the similar collector current and voltage waveforms as shown in Figure 6, where larger value of the shunt capacitance contributes to smaller collector efficiency.

Consider the idealized Class E load network with a shunt capacitance where a quarterwave transmission line is connected between the series inductance and fundamentally tuned series L_0C_0-circuit as shown in Figure 7. Let the output current flowing into the load is sinusoidal given by equation (21) where the phase shift φ is due to the shunt capacitance and series inductance.

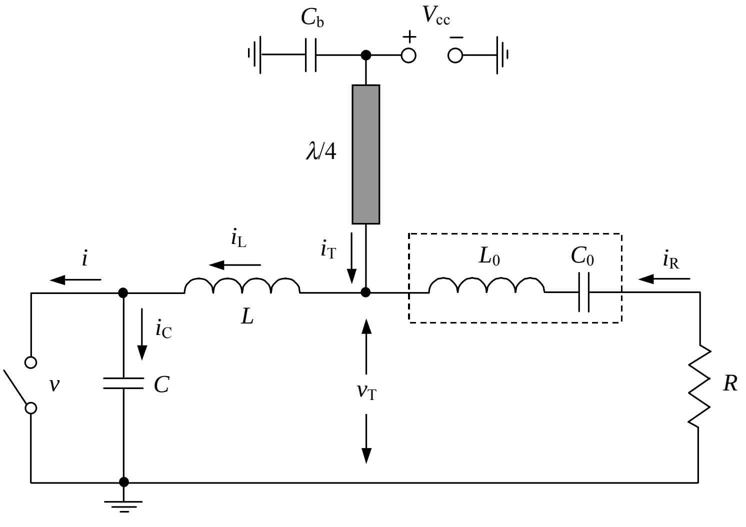


Fig. 7. Equivalent circuit of Class E power amplifier with quarterwave transmission line

When switch is on for $0 \le \omega t < \pi$, the current flowing through the shunt capacitance $i_C(\omega t) = \omega C \frac{dv(\omega t)}{d(\omega t)} = 0$ and, consequently,

$$i(\omega t) = i_L(\omega t) = i_T(\omega t) + i_R(\omega t). \tag{30}$$

When switch is off for $\pi \le \omega t < 2\pi$, there is no current flowing through the switch, i.e., $i(\omega t) = 0$, and the current flowing through the shunt capacitance C is

$$i_C(\omega t) = i_L(\omega t) = i_T(\omega t) + i_R(\omega t). \qquad (31)$$

Using equations (15) and (21) for currents flowing into the transmission line and load, an important relationship can be derived from equations (30) and (31):

$$i_L(\omega t) - i_L(\omega t + \pi) = 2\, i_R(\omega t). \qquad (32)$$

The current $i_L(\omega t + \pi)$ can be expressed through the voltages v_T and $v_L(\omega t) = \omega L \frac{di_L(\omega t)}{d(\omega t)}$ as

$$i_L(\omega t + \pi) = \omega C \frac{d}{d(\omega t)}\left[v_T(\omega t + \pi) - \omega L \frac{di_L(\omega t + \pi)}{d(\omega t)} \right]. \qquad (33)$$

Hence, using equation (14) under switch-on condition when it yields

$$v_T(\omega t + \pi) = 2V_{cc} - \omega L \frac{di_L(\omega t)}{d(\omega t)}, \qquad (34)$$

the resulting second-order differential equation can be obtained from equations (32) and (33) when switch is off for $\pi \le \omega t < 2\pi$ and $i_L = i_C$:

$$\frac{d^2 i_C(\omega t)}{d(\omega t)^2} + \frac{q^2}{2} i_C(\omega t) + I_R \sin(\omega t + \varphi) = 0, \qquad (35)$$

which general solution can be obtained in the form of

$$i_C(\omega t) = C_1 \cos\left(\frac{q}{\sqrt{2}} \omega t \right) + C_2 \sin\left(\frac{q}{\sqrt{2}} \omega t \right) + \frac{2 I_R}{2 - q^2} \sin(\omega t + \varphi), \qquad (36)$$

where $q = 1/\omega\sqrt{LC}$.

The initial off-state conditions to determine the coefficients C_1 and C_2 can be obtained using equations (32) and (34) with taking into account the current continuity between on-state and off-state operation modes:

$$i_C(\omega t)\big|_{\omega t = \pi} = 2 i_R(\pi), \qquad (37)$$

$$\left.\frac{di_C(\omega t)}{d(\omega t)}\right|_{\omega t = \pi} = \frac{V_{cc}}{\omega L} - I_R \cos(\varphi). \qquad (38)$$

Hence, being normalized to the load current amplitude I_R, the coefficiencts C_1 and C_2 can be calculated from

$$\frac{C_1}{I_R} = -\frac{\sqrt{2}}{qp} \sin\left(\frac{q\pi}{\sqrt{2}} \right) - \frac{q\sqrt{2}}{2 - q^2} \sin\left(\frac{q\pi}{\sqrt{2}} \right) \cos\varphi - 2\frac{1 - q^2}{2 - q^2} \cos\left(\frac{q\pi}{\sqrt{2}} \right) \sin\varphi, \qquad (39)$$

$$\frac{C_2}{I_R} = \frac{\sqrt{2}}{qp} \cos\left(\frac{q\pi}{\sqrt{2}} \right) + \frac{q\sqrt{2}}{2 - q^2} \cos\left(\frac{q\pi}{\sqrt{2}} \right) \cos\varphi - 2\frac{1 - q^2}{2 - q^2} \sin\left(\frac{q\pi}{\sqrt{2}} \right) \sin\varphi, \qquad (40)$$

where $p = \omega L I_R / V_{cc}$.

The voltage across the switch during off-state operation is produced by the charging of the shunt capacitance C according to

$$v(\omega t) = \frac{1}{\omega C} \int_{\pi}^{\omega t} i_{\mathrm{C}}(\omega t)\, d\omega t\,. \tag{41}$$

To calculate three unknown parameters q, p and φ from equation (41) using equation (36), it is necessary to use two optimum conditions given by equations (1) and (2) and to add an additional equation defining the supply voltage V_{cc} from Fourier series expansion as

$$V_{\mathrm{cc}} = \frac{1}{2\pi} \int_{0}^{2\pi} v(\omega t)\, d(\omega t). \tag{42}$$

As a result, solving the system of three equations with three unknown parameters numerically gives the following values:

$$q = 1.649, \tag{43}$$

$$p = 1.302, \tag{44}$$

$$\varphi = -40.8°. \tag{45}$$

The DC supply current I_0 as a function of the load current amplitude I_{R} can be found using equations (32) and (36) and Fourier formula by

$$I_0 = \frac{1}{2\pi} \int_{0}^{2\pi} i(\omega t)\, d(\omega t) = 0.482\, I_{\mathrm{R}}\,. \tag{46}$$

In Figure 8, the normalized (*a*) load current, (*b*) collector voltage and (*c*) current waveforms for idealized optimum Class E mode with a quarterwave transmission line are shown. From collector voltage and current waveforms it follows that, when the transistor is turned on, there is no voltage across the switch and the current i consisting of the load sinusoidal current and transmission line current (see Figure 9(*b*)) flows through the switch. However, when the transistor is turned off, this current now flows through the shunt capacitance C (see Figure 9(*a*)). As a result, there is no nonzero voltage and current simultaneously that means a lack of the power losses and gives an idealized collector efficiency of 100%.

The series inductance L and shunt capacitance C using equations (43), (44) and (46) can be obtained, respectively, by

$$L = 1.349 \frac{R}{\omega}, \tag{47}$$

$$C = \frac{0.2725}{\omega R}. \tag{48}$$

The optimum load resistance R for the specified values of the supply voltage V_{cc} and output power P_{out}, taking into account that $R = 2P_{out} / I_R^2$ and $V_{cc}I_0 = P_{out}$ for 100-percent collector efficiency, can be obtained from equations (44) and (46) by

$$R = 0.465 \frac{V_{cc}^2}{P_{out}}. \tag{49}$$

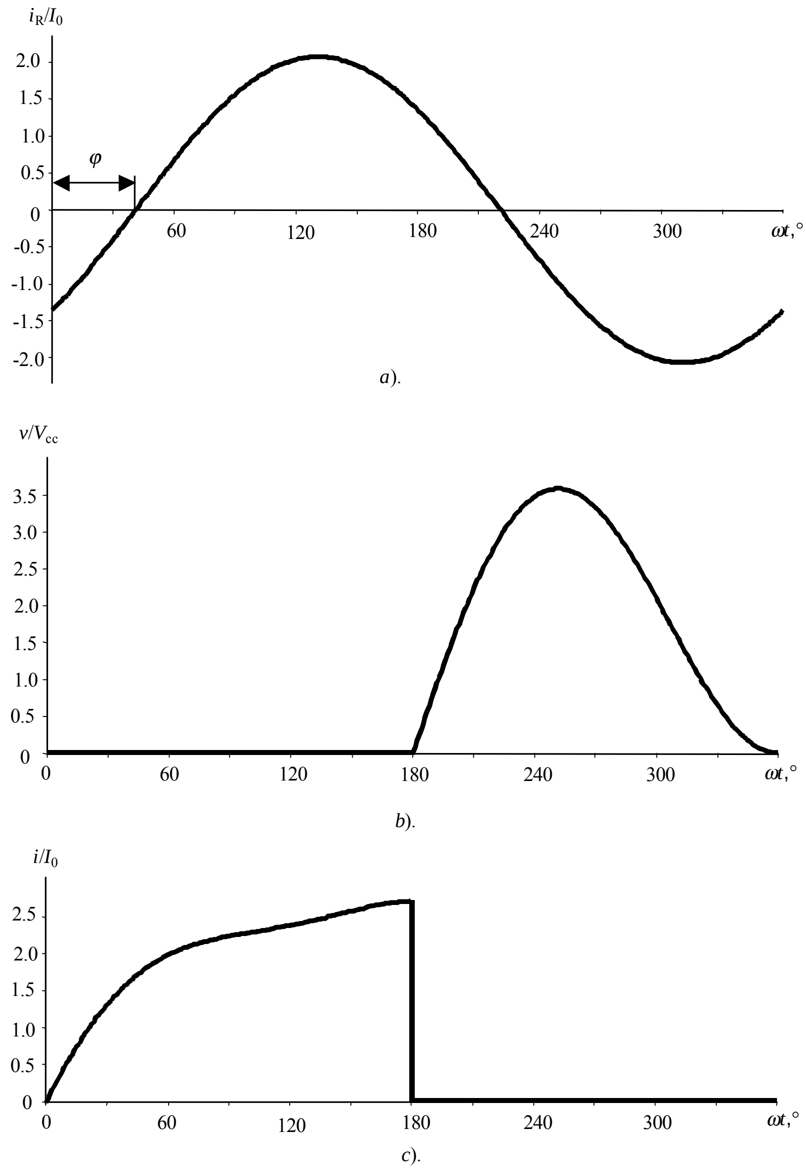


Fig. 8. Voltage and current waveforms of Class E power amplifier with quarter-wave transmission line

The peak collector current I_{max} and peak collector voltage V_{max} can be determined directly from numerical calculation that gives

$$I_{max} = 2.714\, I_0, \tag{50}$$

$$V_{max} = 3.589\, V_{cc}. \tag{51}$$

When designing the optimum Class E operation mode with a quarterwave transmission line, it is very important to know up to which maximum frequency such an efficient operation mode can be extended. In this case, it is required to establish a relationship between maximum operation frequency f_{max}, device output capacitance C_{out} and supply voltage V_{cc}. The device output capacitance C_{out} gives the main limitation of the maximum operation frequency. So, using equation (48) when $C = C_{out}$ gives the value of maximum operation frequency of

$$f_{max} = 0.093 \frac{P_{out}}{C_{out} V_{cc}^2}, \quad (52)$$

which is in 1.63 times greater than that one for the optimum Class E power amplifier with a shunt capacitance [10].

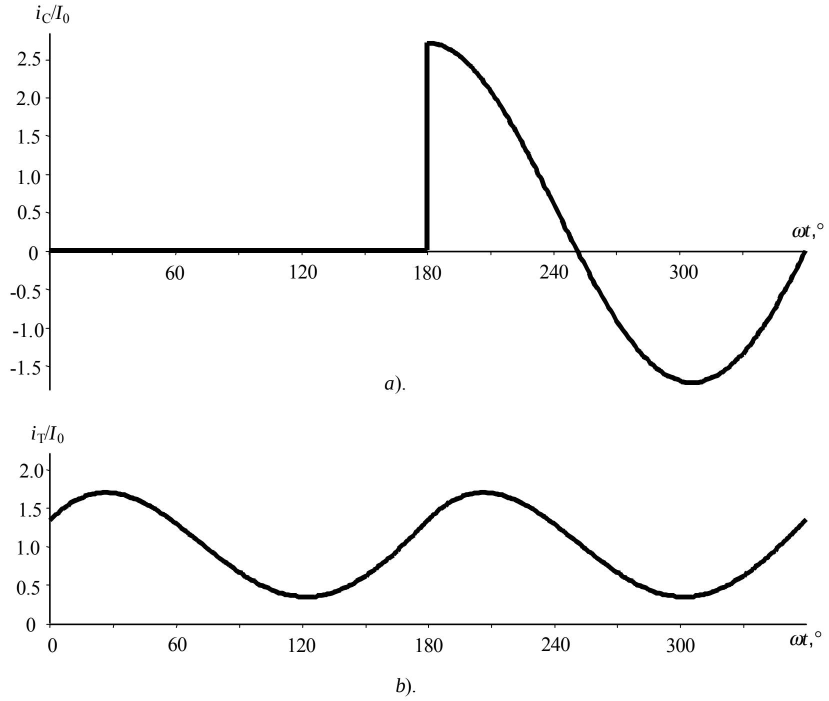
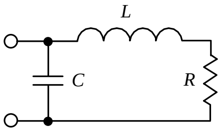
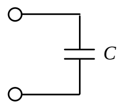
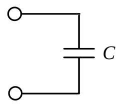
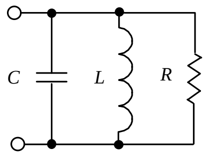
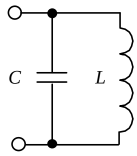
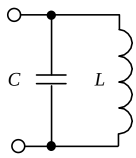
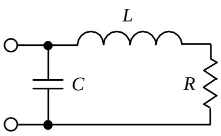
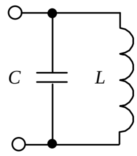
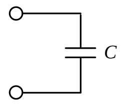


Fig. 9. Current waveforms of Class E power amplifier with transmission line

In Table 1, the optimum impedances seen from the device collector at the fundamental and higher-order harmonic components are illustrated by the appropriate circuit configurations. It can be seen that Class E mode with a quarterwave transmission line shows different impedance properties at even and odd harmonics. At odd harmonics, the optimum impedances can be established by the shunt capacitance as it is required for all harmonics in Class E with a shunt capacitance [2]. At even harmonics, the optimum impedances are realized using a parallel *LC* circuit as it is required for all harmonics in Class E with a parallel circuit [11]. Thus, the frequency properties of a grounded quarterwave transmission line with its open circuit conditions at odd harmonics and short circuit conditions at even harmonics allow Class E with a quarterwave transmission line to combine simultaneously the harmonic impedance conditions typical for both Class E with a shunt capacitance and Class E with a parallel circuit.

Table 1. Optimum impedances at fundamental and harmonics for different Class E load networks

Class E load network	f_0 (fundamental)	$2nf_0$ (even harmonics)	$(2n+1)f_0$ (odd harmonics)
Class E with shunt capacitance [2]			
Class E with parallel circuit [11]			
Class E with quarterwave transmission line			

V. CLASS E LOAD NETWORKS WITH QUARTERWAVE TRANSMISSION LINE

The theoretical results obtained for Class E mode with a quarterwave transmission line show that it is enough to use a very simple load network to realize the optimum impedance conditions even for four harmonics. In this case, as the shunt capacitance C and series inductance L provide optimum inductive impedance at the fundamental and the quarterwave transmission line realizes the shortening of even harmonics, it is only required to provide an open circuit cindition at third harmonic component. In addition, as it follows from Figure 9(b), the current flowing into the transmission line is very closed to the sinusoidal second-harmonic current, which means that the level of forth and higher-order harmonics is negligible because of the significant shunting effect of the capacitance C. Consequently, when the ideal series L_0C_0-circuit is replaced by the output matching circuit, the optimum impedance conditions for Class E load network with a quarterwave transmission line can be practically fully realized by simply providing an open circuit condition at the third harmonic component.

In Figure 10, the circuit schematic of the lumped Class E power amplifier with a quarterwave transmission line is shown, where the paralel L_1C_1 resonant circuit tuned on the third harmonic is used and C_b is the blocking capacitance. Since at the fundamental the reactance of this circuit is inductive, it is enough to use the shunt capacitance C_2 composing the L-type matching circuit to provide the required impedance matching of optimum Class E load resistance R with the standard load impedance of R_L = 50 Ohm. To calculate the parameters of the circuit elements, consider the loaded quality factor $Q_L = \omega C_2 R_L$, which also can be written as

$$Q_L = \sqrt{\left(\frac{R_L}{R}\right) - 1} \tag{53}$$

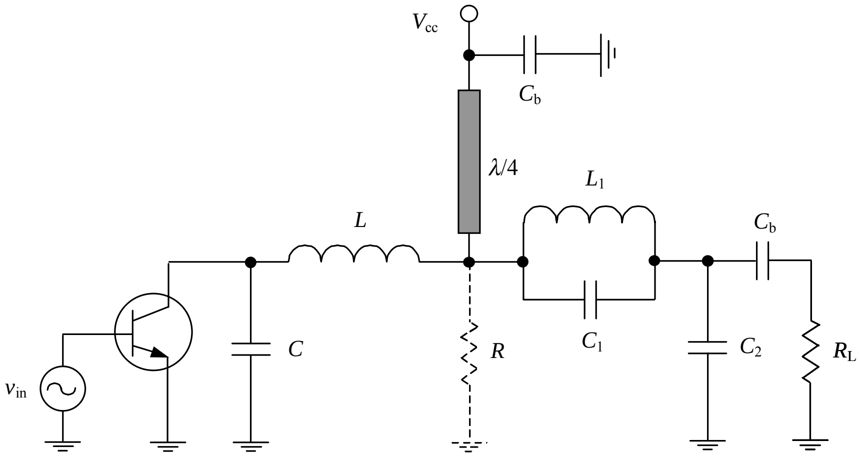


Fig. 10. Schematic of Class E power amplifier with lumped load network

As a result, the matching circuit parameters can be calculated from

$$C_2 = \frac{Q_L}{\omega R_L}, \tag{54}$$

$$L_1 = \frac{8}{9}\frac{Q_L R}{\omega}, \tag{55}$$

$$C_1 = \frac{1}{9\omega^2 L_1}. \tag{56}$$

At microwaves, the series lumped inductance L should be replaced by the short-length series transmission line. In this case, when the shunt capacitance C represents a fully internal active device output capacitance, the bondwire and lead inductances can also be taken into account providing the required inductive reactance and making the series transmission line shorter. In Figure 11, the circuit schematic of the transmission-line Class E power amplifier with a quarterwave transmission line is shown.

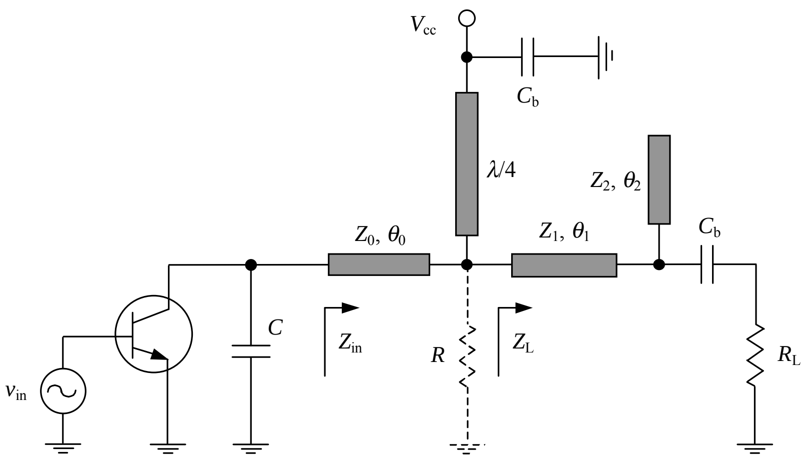


Fig. 11. Schematic of transmission-line Class E power amplifier

Usually the transmission line characteristic impedance Z_0 (in most cases equal to 50 Ohm) is much higher than the required optimum load network resistance R. Consequently, the input impedance of the series transmission line with the characteristic impedance Z_0 and electrical length θ_0 under the condition of $(R \tan\theta_0 / Z_0) << 1$ and sufficiently short transmission line with the electrical length of less than 45 degree is determined by

$$Z_{\text{in}} = Z_0 \frac{R + jZ_0 \tan\theta_0}{Z_0 + jR\tan\theta_0} = Z_0 \frac{\dfrac{R}{Z_0} + j\tan\theta_0}{1 + j\dfrac{R}{Z_0}\tan\theta_0} \cong R + jZ_0 \tan\theta_0. \tag{57}$$

Using equation (47), the required optimum value of θ_0 for Class E mode is obtained from

$$\theta_0 = \tan^{-1}\left(1.349\frac{R}{Z_0}\right). \tag{58}$$

The output matching circuit is necessary to match the required optimum resistance R calculated in accordance with equation (49) to 50-ohm load and also to provide an open circuit condition at the third harmonic component. This can be easily done using the output matching topology in the form of the L-type transformer with series transmission line and open circuit stub. In this case, the electrical lengths of the series transmission line and open circuit stub should be chosen equal to 30 degree each. The load impedance Z_L seen by a quarterwave transmission line can be written by

$$Z_L = Z_1 \frac{R_L\left(Z_2 - Z_1\tan^2\theta\right) + jZ_1Z_2\tan\theta}{Z_1Z_2 + j(Z_1 + Z_2)R_L\tan\theta} \tag{59}$$

where $\theta = \theta_1 = \theta_2 = 30°$, Z_1 and θ_1 are the characteristic impedance and electrical length of the series transmission line, and Z_2 and θ_2 are the characteristic impedance and electrical length of the open circuit stub.

Hence, the conjugate matching with the load can be provided by proper choosing of the characteristic impedances Z_1 and Z_2. Separating equation (59) by the real and imaginary parts, the following system of two equations with two unknown parameters are obtained:

$$\begin{cases} Z_1^2Z_2^2 - R_L^2(Z_1 + Z_2)\left(Z_2 - Z_1\tan^2\theta\right) = 0 \\ (Z_1 + Z_2)^2 R_L^2 R\tan^2\theta - Z_1^2Z_2^2\left[R_L\left(1 + \tan^2\theta\right) - R\right] = 0 \end{cases}, \tag{60}$$

which allows to calculate the characteristic impedances Z_1 and Z_2. This system of two equations can be explicitly solved as a function of the parameter $r = R_L/R$ resulting in

$$\frac{Z_1}{R_L} = \frac{\sqrt{4r-3}}{r}, \tag{61}$$

$$\frac{Z_1}{Z_2} = 3\left(\frac{r-1}{r}\right). \tag{62}$$

Thus, for the specified value of the parameter r with required Class E optimum load resistance R and standard load R_L = 50 Ohm, the characteristic impedance Z_1 is calculated from equation (61). Then, the characteristic impedance Z_2 is calculated from equation (62). For example, if the required optimum load resistance is equal to R = 12.5 Ohm resulting in r = 4, the characteristic impedance of the series transmission line is equal to Z_1 = 45 Ohm and the characteristic impedance of the open circuit stub is equal to Z_2 = 20 Ohm.

Unlike the transmission-line Class E load networks shown in Figure 3(*b*) with two-harmonic control [4] and in Figure 3(*c*) with three-harmonic control [5], the Class E load network with a quarterwave transmission line (see Figure 11), which can provide the optimum impedance conditions at least for four harmonics, is very simple in circuit implementation and doesn't require an additional lumped RF choke element. In addition, there is no need to use the special computer simulation tools required to calculate the parameters of the existing Class E transmission-line load-network topologies [5, 12, 13], as all parameters of the Class E load network with a quarterwave transmission line as well as the output matching circuit parameters are easily calculated explicitly from simple analytical equations. Besides, such a Class E load network with a quarterwave transmission line is very useful in practical design providing simultaneously significant higher-order harmonic suppression.

VI. CONCLUSIONS

The results of exact time domain analysis of the switched-mode tuned Class E power amplifiers with a quarterwave transmission line are presented with analytical determination of the optimum load network parameters. Effects of the quarterwave transmission line and collector capacitance on the current and voltage waveforms are discussed and analyzed. The ideal collector voltage and current waveforms demonstrate a possibility of 100-percent efficiency without overlapping between each other. The ideal current and voltage waveforms for ideal Class F with a quarterwave transmission line are also given. A possibility of the Class E load network implementation including output matching circuit at RF and microwave frequencies using lumped and transmission line elements is considered with accurate derivation of the circuit parameters. The switched-mode Class E power amplifiers with a quarterwave transmission line offer a new challenge for RF and microwave power amplification providing high-efficiency operation conditions.

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