Comprehensible Electromagnetics for Comprehensive Engineering

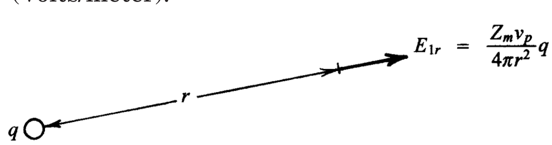
By W. Scott Bennett, Ph.D.

Dr. W. Scott Bennett derives a nanoscale approach to describe the strengths of magnetic and electric near fields that offers an accurate alternative to traditional Hertzian doublet calculations.

Electromagnetic theory has long been based on abstract mathematics; however, to be well-engineered, physical causes and effects must be well understood. That has inspired the cause-based explanations of electromagnetic fields that follow.

Charged Particle Fields

One microscopic particle with an electric charge of q (Coulombs) in an otherwise empty medium causes the *r-directed electric field* E_{1r} (Volts/meter).



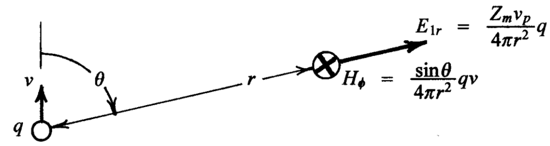
Z_m (Ohms) is the *characteristic impedance* of the medium containing q;

v_p (meters/second) is the *field propagation velocity* in that medium;

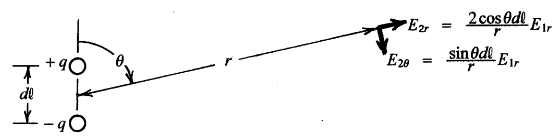
r (meters) is the *distance from* q *to the observation point of* E_{1r} (V/m); and

$4\pi r^2$ is an *imaginary sphere* in which q is centered and on which E_{1r} is observed.

If q moves with a *constant velocity* of v (meters/second), then qv adds to E_{1r} (V/m) the $\phi-$ *directed magnetic field* H_ϕ (Amperes/meter).



An electric dipole is two particles of charge $+q$ and $-q$ separated by a distance $d\ell \rightarrow 0$. And, at any distance $r >> d\ell$ measured from the midpoint of $d\ell$, the charges $+q$ and $-q$ together cause the electric fields E_{2r} and $E_{2\theta}$ (V/m).



These expressions for E_{2r} and $E_{2\theta}$ result, because $d\ell \rightarrow 0$ and $r >> d\ell$ for almost all r. And, when $r >> d\ell$, the propagation distances from $\pm q$ to (r, θ, ϕ) effectively equal $r \mp \cos\theta d\ell/2$. So, the added electric fields of $+q$ and $-q$ can be accurately viewed as

$$E_{2r} = \frac{r^2 E_{1r}}{\left(r - \frac{\cos\theta d\ell}{2}\right)^2} - \frac{r^2 E_{1r}}{\left(r + \frac{\cos\theta d\ell}{2}\right)^2}$$

$$= \frac{\left(r + \frac{\cos\theta d\ell}{2}\right)^2 - \left(r - \frac{\cos\theta d\ell}{2}\right)^2}{\left(r^2 - \left(\frac{\cos\theta d\ell}{2}\right)^2\right)^2} r^2 E_{1r} = \frac{2\cos\theta d\ell}{r} E_{1r} \quad \text{(V/m)}$$

and

$$E_{2\theta} = \frac{\sin\theta d\ell/2}{r - \frac{\cos\theta d\ell}{2}} E_{1r} + \frac{\sin\theta d\ell/2}{r + \frac{\cos\theta d\ell}{2}} E_{1r}$$

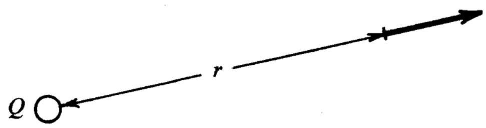
$$= \frac{\left(r + \frac{\cos\theta d\ell}{2}\right) + \left(r - \frac{\cos\theta d\ell}{2}\right)}{r^2 - \left(\frac{\cos\theta d\ell}{2}\right)^2} \frac{\sin\theta d\ell}{2} = \frac{\sin\theta d\ell}{r} E_{1r} \quad \text{(V/m)}$$

because

$$\left(\frac{\cos\theta d\ell}{2}\right)^2 \leq \left(\frac{d\ell}{2}\right)^2 \quad \text{and} \quad \left(\frac{d\ell}{2}\right)^2 \cong 0$$

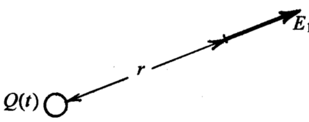
Point Source Fields

A stationary volume dV *containing numerous microscopic particles* that have a *net electric charge* of Q (C) causes the *electric field* E_{1r} (V/m).



$$E_{1r} = \frac{Z_m v_p}{4\pi r^2} Q$$

A *time-varying net charge* of *Q(t)* (C) in *dV* will cause the *two-component electric field* $E_{1r}(\tau) + \underline{E}_{1r}(\tau)$ (V/m).

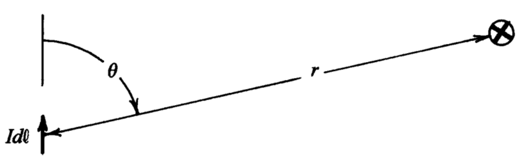


$$E_{1r}(\tau) + \underline{E}_{1r}(\tau) = \frac{Z_m v_p}{4\pi r^2}\left[Q(\tau) + t_p \frac{dQ(\tau)}{d\tau}\right]$$

$\tau = t - t_p$ is *field cause time, t* is *field observation time,* and

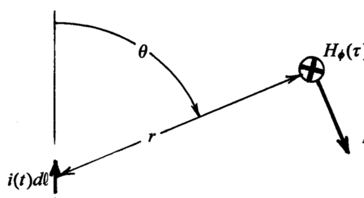
$t_p = r/V_p$ is *field propagation time.*

If some of the particles in *dV* are *moving* in the direction of its *length dℓ* , a *steady movement of net charge, Idℓ* (Axm), causes the *magnetic field* H_ϕ (A/m).



$$H_\phi = \frac{\sin\theta}{4\pi r^2} I d\ell$$

A time-varying charge movement i(t)dℓ (Axm) causes the *two-component magnetic field* $H_\phi(\tau) + \underline{H}_\phi(\tau)$ (A/m) and the *electric field* $\underline{E}_\theta(\tau) = Z_m H_\phi(\tau)$ (V/m).



$$H_\phi(\tau) + \underline{H}_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] d\ell$$

$$\underline{E}_\theta(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}\left[t_p \frac{di(\tau)}{d\tau}\right] d\ell$$

A <u>*nanocurrent i(t)dℓ*</u> is a volume *dV* of length *dℓ* and cross-sectional area *dA,* that contains *both* the *moving charge i(t)dℓ* (Axm), and the *net charge*

$$Q(t) = \int_0^t (i_{in}(t) - i_{out}(t))dt = d\ell \int \frac{di(t)}{d\ell} dt$$

$$= \frac{d\ell}{v_{pc}} \int di(t) = \frac{i(t)d\ell}{v_{pc}} \qquad \text{(C)}$$

where $v_{pc} = d\ell/dt$ (m/s) is the field propagation velocity in *dV*.

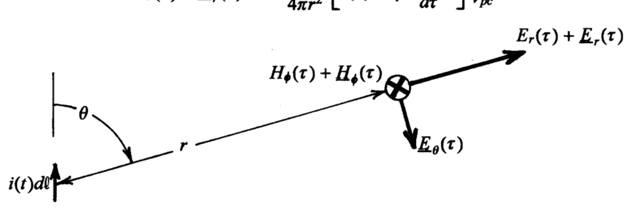
Therefore, a *nanocurrent i(t)dℓ has the point-source fields*

$$H_\phi(\tau) + \underline{H}_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(A/m)}$$

$$\underline{E}_\theta(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}\left[t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(V/m)}$$

and

$$E_r(\tau) + \underline{E}_r(\tau) = \frac{Z_m v_p}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] \frac{d\ell}{v_{pc}} \qquad \text{(V/m)}$$



The Hertzian Dipole and Its Fields

A Hertzian dipole would be two nanocurrents, one that has end-currents of 0 and - *i(t)* , and the other with end-currents of *i(t)* and 0. So, *an isolated Hertzian dipole* would have *net charges* of

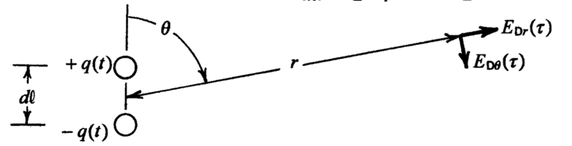
$$\int (0 - i(t))dt = -q(t) \quad \text{and} \quad \int (i(t) - 0)dt = +q(t) \qquad \text{(C)}$$

separated by *dℓ,* and the fields of *+q(t)* and *– q(t)* would add the same as the fields of *+q* and *–q* of an electric dipole, to be

$$E_{Dr}(\tau) = \frac{2\cos\theta d\ell}{r}[E_{1r}(\tau) + \underline{E}_{1r}(\tau)] = 2\frac{Z_m \cos\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau)\right] d\ell \qquad \text{(V/m)}$$

and

$$E_{D\theta}(\tau) = \frac{\sin\theta d\ell}{r}[E_{1r}(\tau) + \underline{E}_{1r}(\tau)] = \frac{Z_m \sin\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau)\right] d\ell \qquad \text{(V/m)}$$

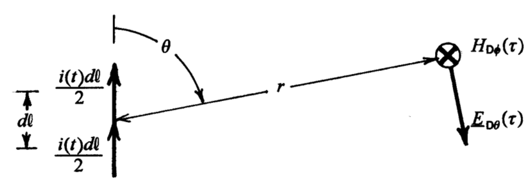


The fields of moving charge would be those of two adjacent nanocurrents, each with an average current of *i(t)/2* and length *dℓ*. Therefore, because *dℓ* → 0, for all *r* >> *dℓ* the fields of moving charge would equal those of one nanocurrent *i(t)dℓ.*

$$H_{D\phi}(\tau) = 2\left[\frac{H_\phi(\tau)}{2} + \frac{\underline{H}_\phi(\tau)}{2}\right] d\ell = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(A/m)}$$

and

$$\underline{E}_{D\theta}(\tau) = \frac{E_\theta(\tau)}{2} + \frac{\underline{E}_\theta(\tau)}{2} = \frac{Z_m \sin\theta}{4\pi r^2}\left[t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(V/m)}$$



Thus, a*n isolated Hertzian dipole's fields* would be

$$E_{Dr}(\tau) = \frac{2Z_m \cos\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau)\right] d\ell \qquad \text{(V/m)}$$

$$H_{D\phi}(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(A/m)}$$

and

$$E_{D\theta}(\tau) + \underline{E}_{D\theta}(\tau) = \frac{Z_m \sin\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau) + t_p \frac{di(\tau)}{d\tau}\right] d\ell \qquad \text{(V/m)}$$

These field equations can be related to textbook equations [1 – 6] for the fields of a Hertzian dipole, as follows. If μ is the *permeability* of the medium *containing i(t)dℓ,* and ϵ is the *permittivity* of that medium, then

$$Z_m = \sqrt{\frac{\mu}{\epsilon}} \quad \text{(Ohms)} \qquad \text{and} \qquad v_p = \frac{1}{\sqrt{\mu\epsilon}} \quad \left(\frac{\text{meters}}{\text{second}}\right)$$

And, if $i(t)$ is assumed to be a sinusoidal current, then as in [1], for example, the replacements

$$i(\tau) \leftarrow I_0 e^{j\omega(t-r/c)}, \quad q(\tau) \leftarrow \frac{I_0 e^{j\omega(t-r/c)}}{j\omega}, \quad \frac{di(\tau)}{d\tau} \leftarrow j\omega I_0 e^{j\omega(t-r/c)}$$

and

$$d\ell \leftarrow L, \quad v_p \leftarrow c, \quad t_p \leftarrow \frac{r}{c}, \quad \text{and} \quad Z_m \leftarrow \frac{1}{\epsilon c}$$

make

$$E_{Dr}(\tau) = Z_m \frac{2\cos\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau)\right]d\ell$$

$$= \frac{1}{\epsilon c}\frac{2\cos\theta}{4\pi r^2}\left[\frac{I_0 e^{j\omega\tau}}{j\omega r/c} + I_0 e^{j\omega\tau}\right]L$$

$$= \frac{I_0 e^{j\omega(t-r/c)} L\cos\theta}{2\pi\epsilon}\left[\frac{1}{cr^2} + \frac{1}{j\omega r^3}\right] = E_r \qquad \text{(V/m)}$$

and

$$H_{D\phi}(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p\frac{di(\tau)}{d\tau}\right]d\ell$$

$$= \frac{\sin\theta}{4\pi r^2}\left[I_0 e^{j\omega\tau} + \frac{r}{c} j\omega I_0 e^{j\omega\tau}\right]L$$

$$= \frac{I_0 e^{j\omega(t-r/c)} L\sin\theta}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right] = H_\phi \qquad \text{(A/m)}$$

and

$$E_{D\theta}(\tau) + E_{D\theta}(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}\left[\frac{q(\tau)}{t_p} + i(\tau) + t_p\frac{di(\tau)}{d\tau}\right]d\ell$$

$$= \frac{1}{\epsilon c}\frac{\sin\theta}{4\pi r^2}\left[\frac{I_0 e^{j\omega\tau}}{j\omega r/c} + I_0 e^{j\omega\tau} + \frac{r}{c} j\omega I_0 e^{j\omega\tau}\right]L$$

$$= \frac{I_0 e^{j\omega(t-r/c)} L\sin\theta}{4\pi\epsilon}\left[\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right] = E_\theta \qquad \text{(V/m)}$$

Conclusion

The *moving charge* $Id\ell$ of a *nanocurrent* $Id\ell$ causes the *magnetic field*

$$H_\phi = \frac{\sin\theta}{4\pi r^2} Id\ell \qquad \text{(A/m)}$$

And, the *moving charge* $Id\ell$ also causes the *net charge* $Id\ell/v_{pc}$, which is the cause of the *electric field*

$$E_r = \frac{Z_m v_p}{4\pi r^2}\frac{Id\ell}{v_{pc}} \qquad \text{(V/m)}$$

A *time-varying nanocurrent* $i(t)d\ell$ causes the *magnetic field*

$$\text{(1)} \qquad H_\phi(\tau) = \frac{\sin\theta}{4\pi r^2}\left[i(\tau) + t_p\frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(A/m)}$$

and the *electric field*

$$\text{(2)} \qquad E_\theta(\tau) = Z_m \frac{\sin\theta}{4\pi r^2}\left[t_p\frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

Its *net charge* $i(t)d\ell/v_{pc}$ causes the *electric field*

$$E_r(\tau) = \frac{Z_m k}{4\pi r^2}\left[i(\tau) + t_p\frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

where $k = v_p / v_{pc}$. And, <u>in very many, if not all, important cases *k =1, and*</u>

$$\text{(3)} \qquad E_r(\tau) = \frac{Z_m}{4\pi r^2}\left[i(\tau) + t_p\frac{di(\tau)}{d\tau}\right]d\ell \qquad \text{(V/m)}$$

The velocities v_p and v_{pc} will be equal when the medium containing $i(t)d\ell$ is either free space, a vacuum, or air, and the $i(t)$ conductor is copper, lead, aluminum, silver, or gold, for example. That follows, because their relative permeabilities and relative permittivities are all equal to 1.

Based on [1 – 7], on [8], and all of the above – especially on equations (1), (2) and (3) immediately above – it is quite clear that nanocurrents should replace Hertzian dipoles as current elements. The result will undoubtedly be more accurate near-field computations and more comprehensible engineering electromagnetics.

References

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About the Author

W. Scott Bennett, Ph.D., served as Assistant Professor at Virginia Polytechnic Institute, where he taught electromagnetics and computer design. He later worked at Hewlett-Packard Company where for 16 years he designed computers and made those designs electromagnetically compatible. Since retiring he has worked to rid basic electromagnetics of abstract mathematics and thus make it easier to understand. He can be reached at: w.scottbennett@juno.com.